



flexMIRT<sup>®</sup>: Flexible Multilevel  
Multidimensional Item Analysis and Test  
Scoring

User's Manual  
Version 2.0

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# Contents

<b>1</b>	<b>Overview</b>	<b>1</b>
<b>2</b>	<b>Basic Calibration and Scoring</b>	<b>3</b>
2.1	Overview . . . . .	3
2.2	Single-Group 2PL Model Calibration . . . . .	5
2.3	Single-Group 3PL Model Calibration . . . . .	12
2.4	Single-Group 3PL EAP Scoring . . . . .	15
2.5	Single-Group 3PL ML Scoring . . . . .	19
2.6	Graded Model Calibration and Scoring . . . . .	20
2.7	1PL Model Fitted to Response Pattern Data . . . . .	23
2.8	The Nominal Categories Model . . . . .	29
2.8.1	Nominal Model . . . . .	29
2.8.2	Generalized Partial Credit Model . . . . .	36
2.8.3	Rating Scale Model . . . . .	37
<b>3</b>	<b>Advanced Modeling</b>	<b>40</b>
3.1	Multiple Group Calibration . . . . .	40
3.2	Characterizing Non-Normal Population Distributions . . . . .	42
3.3	Multiple Group Scoring Only . . . . .	43
3.4	DIF Analyses . . . . .	45
3.5	Multiple Group Multilevel Model . . . . .	49
3.6	Multidimensional Model - 4 Correlated Factors . . . . .	53
3.7	The Bifactor Model . . . . .	56
3.7.1	Bifactor Model Using the Reparameterized Nominal Categories Model . . . . .	56
3.7.2	Bifactor Model Using the 2PL . . . . .	59
<b>4</b>	<b>Simulation</b>	<b>61</b>
4.1	A Syntax-Only Example Using Value Statement . . . . .	61
4.2	The First Example Redone Using Parameter File . . . . .	64

4.3	The Parameter File Layout . . . . .	65
4.3.1	Column Progression for Groups . . . . .	66
4.3.2	Column Progression for Items . . . . .	67
4.4	A 3PL Model Simulation Example . . . . .	68
4.5	Simulating Data with Non-Normal Theta Distributions . . . . .	71
4.6	Simulating Bifactor Model Data . . . . .	72
4.7	Simulating Group Differences . . . . .	74
4.8	Multilevel Bifactor Model Simulation . . . . .	77
4.9	Internal Simulation Function . . . . .	80
4.9.1	Accessing the Internal Simulation Function . . . . .	80
4.9.2	The Simulation Output File . . . . .	82
<b>5</b>	<b>Metropolis-Hastings Robbins-Monro Estimation for High Dimensional Models</b>	<b>85</b>
5.1	Overview of MH-RM Estimation . . . . .	85
5.2	MH-RM Specific Syntax Options . . . . .	87
5.3	Examples Utilizing MH-RM Estimation . . . . .	89
5.3.1	MIRT Model with Correlated Factors . . . . .	89
5.3.2	Multi-level MIRT Model . . . . .	93
5.4	Fixed Effects Calibration . . . . .	97
<b>6</b>	<b>Cognitive Diagnostic Models</b>	<b>99</b>
6.1	General CDM Modeling Framework . . . . .	100
6.2	CDM Specific Syntax Options . . . . .	101
6.3	CDM Modeling Examples . . . . .	102
6.3.1	Introductory CDM Examples . . . . .	103
6.3.2	Replicating Published Examples . . . . .	123
<b>7</b>	<b>Details of the Syntax</b>	<b>138</b>
7.1	The Project Section . . . . .	138
7.2	The Options Section . . . . .	138
7.3	The Groups Section . . . . .	151
7.4	The Constraints Section . . . . .	157
<b>8</b>	<b>Models</b>	<b>162</b>
8.1	A Generalized Formulation . . . . .	162
8.2	Item Response Models . . . . .	163
8.2.1	Model with Pseudo-Guessing Parameter . . . . .	163
8.2.2	Graded Response Model . . . . .	164

8.2.3 Nominal Categories Model . . . . .	164
<b>Appendices</b>	<b>166</b>
<b>A Quick-Start Installation and User Guide</b>	<b>167</b>
A.1 Installing flexMIRT . . . . .	167
A.2 Using flexMIRT . . . . .	172
<b>B flexMIRT via the Command Line Interface</b>	<b>174</b>
B.1 flexMIRT via the CLI - single file run . . . . .	174
B.2 flexMIRT via the CLI - batch mode . . . . .	177
<b>C Plotting</b>	<b>180</b>
C.1 Plotting ICCs, TCCs, and Information Functions . . . . .	180
<b>D Labeled Output Files</b>	<b>184</b>
<b>E Licensing Information</b>	<b>188</b>
E.1 Commercial use of flexMIRT . . . . .	188
E.2 Classroom Discounts for flexMIRT . . . . .	188
<b>References</b>	<b>189</b>
<b>Index</b>	<b>194</b>

# List of Tables

4.1	Simulation - Graded Model PRM File Content . . . . .	65
4.2	Key Codes for Columns With Set Options . . . . .	66
4.3	Simulation - 3PL Parameter File with Labeled Columns . . . . .	70
4.4	Labeled Parameter File for Bifactor Model Simulation . . . . .	73
4.5	Simulating DIF and Group Mean Differences - Parameter Values	76
6.1	Q-matrix for Basic CDM Demonstrations . . . . .	103
6.2	Model Fit for Basic CMDs Fit to Simulated Data . . . . .	119
6.3	Q-matrix for Testlet-DINA Example . . . . .	120
6.4	Q-matrix for Rupp, Templin, & Henson's Example 9.2 . . . . .	123
6.5	Generating latent class mean values and flexMIRT® and MPLUS estimates for Rupp, Templin, & Henson's Example 9.2 . . . . .	128
6.6	Guessing and Slippage Parameters from flexMIRT and MCMC	137
7.1	Constraint Commands and Keywords . . . . .	157
D.1	Labeled -sco file - EAP scores, 1 Factor . . . . .	184
D.2	Labeled -sco file - MAP scores, 1 Factor . . . . .	184
D.3	Labeled -sco file - EAP scores, 2 Factors . . . . .	185
D.4	Labeled -sco file - EAP scores, 2 Factors with User-supplied ID Variable . . . . .	185
D.5	Labeled -sco file - MAP scores, 2 Factors . . . . .	185
D.6	Labeled -prm file - GRM with 1 Factor, 1 Group . . . . .	186
D.7	Labeled -prm file - 2PL with 6 Factors, 1 Group . . . . .	186
D.8	Labeled -prm File - 3PL with 1 Factor, 3 groups . . . . .	187

# List of Examples

2.1	2PL Calibration Syntax . . . . .	5
2.2	Single Group 3PL Calibration Syntax . . . . .	12
2.3	3PL EAP Scoring example . . . . .	16
2.4	3PL ML Scoring example . . . . .	19
2.5	Graded Model Combined Calibration and Scoring Syntax with Recoding . . . . .	20
2.6	1PL Calibration and Scoring with Response Pattern Data . .	24
2.7	Nominal Model Calibration . . . . .	29
2.8	Nominal Model Calibration 2 . . . . .	32
2.9	Generalized Partial Credit Model . . . . .	36
2.10	Rating Scale Model with Trend Contrasts . . . . .	37
2.11	Rating Scale Model with Triangle Contrasts . . . . .	38
3.1	2-Group 3PL Using Normal Priors with EAP Scoring . . . . .	40
3.2	2-Group 3PL with Empirical Histogram (Excerpts) . . . . .	42
3.3	2-Group 3PL MAP Scoring from Parameter Estimates . . . . .	44
3.4	2-Group 3PL Summed Score to EAP Conversion Table . . . . .	44
3.5	DIF: Test All Items (Excerpts) . . . . .	45
3.6	DIF: Test Candidate Items (Excerpts) . . . . .	48
3.7	Multiple Group Multilevel Model (Excerpts) . . . . .	50
3.8	Four Factor GRM Model with Correlated Factors . . . . .	53
3.9	Bifactor Structure - Nominal Model . . . . .	57
3.10	Bifactor Structure - 2PL . . . . .	59
4.1	Simulation - Graded Model Syntax Only . . . . .	62
4.2	Simulation - Graded Model Using PRM File . . . . .	64
4.3	Simulation - 3PL Model Normal Population . . . . .	69
4.4	Simulation - Empirical Histogram Prior . . . . .	71
4.5	Simulation - Bifactor Structure From Existing Parameter File	72
4.6	Simulation - DIF and Group Mean Differences . . . . .	75
4.7	Simulation - Multilevel Model with Bifactor Structure . . . . .	77

5.1	<Options> - Engine Mode Settings to Call MH-RM Estimation and MH-RM Specific Options . . . . .	87
5.2	MHRM: 4 Factor MIRT Model . . . . .	89
5.3	MHRM: Two-level MIRT Model . . . . .	93
5.4	Fixed Effects Calibration . . . . .	97
6.1	<Options>, <Groups>, and <Constraints>- CDM Specific Options . . . . .	101
6.2	C-RUM Model Fit To Simulated Data . . . . .	104
6.3	DINA Fit to Simulated Data . . . . .	111
6.4	DINO Fit to Simulated Data . . . . .	115
6.5	Testlet Structure with DINA model . . . . .	121
6.6	Rupp, Templin, & Henson - Example 9.2 . . . . .	123
6.7	Rupp, Templin, & Henson - Example 9.2 without D group . .	131
6.8	de la Torre Subtraction Syntax . . . . .	134
7.1	<Project> section commands . . . . .	138
7.2	<Options> - Engine Mode and Output Statements . . . . .	139
7.3	<Options> - Technical Specifications . . . . .	141
7.4	<Options> - GOF and LD Indices Commands . . . . .	145
7.5	<Options> - DIF Analysis Commands . . . . .	148
7.6	<Options> - MH-RM Specific Options . . . . .	149
7.7	<Groups> - General Data/Model Descriptors . . . . .	151
7.8	<Groups> - Specialized Data/Model Descriptors . . . . .	154
7.9	Constraints Across Groups, Specifying a Prior Distribution, and Freely Estimating Theta . . . . .	159
C.1	R plotting code . . . . .	181

# List of Output

2.1	2PL Calibration Output - Summary and Controls . . . . .	7
2.2	2PL Calibration Output - Item and Group Parameters . . . . .	9
2.3	2PL Calibration Output - Information Functions . . . . .	10
2.4	2PLM Calibration Output - Goodness of Fit Indices . . . . .	11
2.5	Single Group 3PL Calibration Output- Item Parameters . . . . .	14
2.6	Single Group 3PL Calibration Output- Extended GOF Table . . . . .	15
2.7	3PL EAP Scoring Output . . . . .	17
2.8	3PL ML Scoring Output . . . . .	20
2.9	Graded Model Calibration - Slopes and Intercepts . . . . .	22
2.10	Graded Model Calibration - Slopes and Thresholds . . . . .	22
2.11	Graded Model Calibration - Extended GOF Table . . . . .	23
2.12	1PL Grouped Data Output - Item Parameters and Factor Loadings . . . . .	26
2.13	1PL Grouped Data Output - Item Fit Diagnostics . . . . .	27
2.14	1PL Grouped Data Output - Summed Score to Scale Score Conversion . . . . .	27
2.15	1PL Grouped Data Output - JSI matrix . . . . .	28
2.16	1PL Grouped Data Output - Haberman Residual Table (Excerpt) . . . . .	28
2.17	Nominal Model Output - Item Parameters . . . . .	31
2.18	Nominal Model 2 Output - Item Parameters . . . . .	34
2.19	Nominal Model 2 Output - GOF values . . . . .	35
3.1	DIF Analyses Output Table . . . . .	47
3.2	Two-Level Model - Means and Variances . . . . .	52
3.3	Correlated Four Factor Model Output - Loadings, Means and Variances . . . . .	55
3.4	Nominal Model Scoring Function Estimates - Excerpt . . . . .	58
4.1	Simulation Control Output - Graded Model Syntax Only . . . . .	63
4.2	Simulated Data Set - Graded Model Syntax Only . . . . .	64
4.3	Simulated Data Set - Graded Model Using PRM File . . . . .	65
4.4	Simulation Control Output for Multilevel Bifactor Model . . . . .	78

4.5	Beginning Columns of Output from 5 Replications . . . . .	82
4.6	Item Parameters with Parameter Number Labels . . . . .	82
4.7	ISF output - Switch from Model 1 to Model 2 Values . . . . .	83
5.1	Correlated Four Factor Model Output - Processing Pane Output	91
5.2	Correlated Four Factor Model Output - Loadings, Means and Variances . . . . .	92
5.3	Two-Level MIRT Model - Processing Pane Output . . . . .	95
5.4	Multilevel MIRT Model - Item and Group Parameters . . . . .	96
5.5	Fixed Effects Calibration Output - Item Parameters . . . . .	98
6.1	C-RUM Output - Item Parameters and Attribute Profile Pos- terior Probabilities . . . . .	107
6.2	C-RUM Output -SCO Output . . . . .	109
6.3	DINA output - Item Parameter Estimates . . . . .	113
6.4	DINA Summed Score Conversion Tables from -ssc File . . . . .	114
6.5	DINO Output - Item Parameters . . . . .	118
6.6	Rupp, Templin, Henson Output - Item Parameters . . . . .	127
6.7	Rupp, Templin, Henson Output - -sco File . . . . .	129
6.8	Rupp, Templin, Henson Output - Item Parameters without D group . . . . .	133

## CHAPTER 1

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### Overview

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Vector Psychometric Group, LLC, a North Carolina-based psychometric software and consulting company, is pleased to announce the immediate availability of flexMIRT<sup>®</sup> Version 2.0, a substantial update to our multilevel and multiple-group item response theory (IRT) software package for item analysis and test scoring. flexMIRT<sup>®</sup> fits a variety of unidimensional and multidimensional IRT models to single-level and multilevel data using maximum marginal likelihood or modal Bayes via Bock-Aitkin EM (with generalized dimension reduction) or MH-RM estimation algorithms. The generalized dimension reduction EM algorithm, coupled with arbitrary user-defined parameter constraints, makes flexMIRT<sup>®</sup> one of the most flexible IRT software programs on the market today and the MH-RM algorithm allows users to efficiently estimate high-dimensional models. flexMIRT<sup>®</sup> produces IRT scale scores using maximum likelihood (ML), maximum a posteriori (MAP), and expected a posteriori (EAP) estimation. It (optionally) produces summed-score to IRT scale score (EAP) conversion tables for unidimensional and multidimensional IRT models. As for the item types, flexMIRT<sup>®</sup> can estimate any arbitrary mixtures of 3-parameter logistic (3PL) model, logistic graded response model (which includes 2PL and 1PL as special cases), and the nominal categories model (including any of its restricted sub-models such as generalized partial credit model, partial credit model, and rating scale model) for both single-level and multilevel data, in any number of groups.

flexMIRT<sup>®</sup> also has some of the richest psychometric and statistical features. New to version 2.0, we have implemented features that allows users to fit Cognitive Diagnostic Models (CDMs). In addition to fully implementing many recently developed multilevel and multidimensional IRT models, flexMIRT<sup>®</sup> supports numerous methods for estimating item parameter standard errors: Supplemented EM, empirical cross-product approximation, Fisher (expected) information matrix, Richardson extrapolation, forward difference,

and sandwich covariance matrix. A multitude of model fit statistics for dimensionality analysis, item-fit testing, and latent variable normality diagnosis are included in flexMIRT<sup>®</sup>. Its multiple-group estimation features easily facilitate studies involving differential item function (DIF) and test linking (including vertical scaling).

A key innovation in flexMIRT<sup>®</sup> is its ability to relax the ubiquitous multivariate normality assumption made in virtually all multidimensional IRT models. With an extended dimension reduction algorithm, it supports the non-parametric estimation of latent density shapes using empirical histograms for both unidimensional and hierarchical (e.g., bifactor and testlet response theory) item factor models in any number of groups, with support for constraints on group means and variances. Finally, it has a full-featured built-in Monte Carlo simulation module that can generate data from all models implemented in flexMIRT<sup>®</sup>.

flexMIRT<sup>®</sup> is easy to use. It has an intuitive syntax. It can import data natively in space-, comma-, or tab- delimited formats. Windows-based flexMIRT<sup>®</sup>, with a friendly graphical user interface (GUI), is available in both 32-bit and 64-bit flavors. flexMIRT<sup>®</sup> is designed with cross-platform compatibility and large-scale production use from the beginning. The computational engine of flexMIRT<sup>®</sup> is written using standard C++, which ensures that it can run on any platform where a C++ compiler exists. A newly-designed memory allocation scheme helps flexMIRT<sup>®</sup> efficiently handle thousands of items and millions of respondents, with no imposed upper limit on the size of the problem.

flexMIRT<sup>®</sup> is fast. For multidimensional and multilevel models that permit dimension reduction, flexMIRT<sup>®</sup> automatically reduces the amount of quadrature computation to achieve a dramatic reduction in computational time. As modern CPU architecture trends toward multi-core design, flexMIRT<sup>®</sup> implements two ways to use multi-threading to further speed up computations by spreading the load automatically to the multiple available cores or processing units.

This user's manual is intended both as a guide to those trying to navigate the complex features implemented in flexMIRT<sup>®</sup> for the first time, and as a reference source. It is assumed that the user is familiar with the theory and applications of IRT. Please visit the website <http://www.flexMIRT.com> for product updates, technical support, and feedback.

## CHAPTER 2

---

### Basic Calibration and Scoring

---

We begin by discussing the basic structure of flexMIRT<sup>®</sup> syntax. We provide examples that demonstrate some standard IRT analyses, such as calibration and scoring. Syntax files for flexMIRT<sup>®</sup> may be created in the GUI by selecting “New” or created using a flat text program (e.g., Notepad). If “New” is selected within the flexMIRT<sup>®</sup> GUI, a command file containing the skeleton code for a basic analysis opens. Once you have modified this command file, it must be saved prior to being able to submit the analysis to flexMIRT<sup>®</sup>; the “Run” button is disabled until the file has been saved to prevent the skeleton code from being over-written. The file name extension for flexMIRT<sup>®</sup> syntax files is \*.flexmirt. Data files must use space, tab, or comma delimiters between variables and may not contain a variable name header. Data should be arranged so columns represent items and each observation has its own row in the dataset. Although flexMIRT<sup>®</sup> has some data manipulation capabilities (recoding, scoring using a provided answer key, etc), it is expected that the user has completed necessary pre-analyses (such as dimensionality tests, ensuring that all response options have observations, etc.) prior to submitting data to the program. Missing data are indicated by -9 by default, although the missing data code may be modified to any numeric value between -127 and 127. All syntax and data files used in this manual are available in the Support section on the flexMIRT<sup>®</sup> website (<http://www.flexMIRT.com/Support>).

---

#### 2.1. Overview

Generally speaking, flexMIRT<sup>®</sup> syntax files are divided into four required sections, each of which provides instructions for distinct parts of the analysis. The four sections, which will be discussed in more detail, are

1. **<Project>** – where you provide a title and a general description of the analysis for record-keeping.

2. **<Options>** – where the type of analysis and options of the analysis are specified.
3. **<Groups>** – where the input data and the IRT models are specified.
4. **<Constraints>** – where parameter constraints (such as equality or fixed parameter values) or univariate priors may be specified.

All four section headers must be present in every command file (e.g., even if no constraints are applied to the parameters, the **<Constraints>** section header must still be declared in the syntax). The other requirement is that all statements in the flexMIRT<sup>®</sup> syntax file must end with a semi-colon. Finally, the commands in the syntax are not case-sensitive (but file names on certain operating systems may be).

Within the **<Project>** section, there are two required commands, **Title = " "**; and **Description = " "**;. The name of the title and the contents of the description must be enclosed in double quotation marks.

The **<Options>** section is where the more technical aspects of the analysis are specified. It is in this section that analysis details such as convergence criteria, scoring methods, standard error estimation methods, or requests for various fit indices may be specified. The only required command is **Mode = ;**, where the user is asked to specify what type of analysis will be conducted. The three options available are **Calibration**, **Scoring**, and **Simulation**. Depending on the **Mode** of the analysis, there may be additional required commands, such as the type of IRT scoring to be conducted in a **Scoring** analysis. These details will be covered as they emerge in the examples.

As noted previously, **<Groups>** is the section where information regarding the groups of item response data is provided. This includes the name(s) of the file(s) containing the data set(s), the number of items, the number of groups and participants, and the number of response options for each item, among other things. Following the **<Groups>** section header, a group name must be provided for the first group (even if there is only one group). The group name is enclosed in percent signs (%) and should be something easy to remember (e.g., **%Group1%** or **%Grade4%** or **%Female%** or **%US%**). The group names become important in specifying constraints in multiple-group analyses. The name is arbitrary, but spaces are not allowed when specifying group names. After a group name has been given, the data file name where the item responses are located is specified, as well as the number of participants in the group, variable names, the number of possible response categories for each item, and the IRT model to be estimated (e.g., **threePL**, **Graded(5)**, etc).

The final section, `<Constraints>`, is reserved for specifying constraints and prior distributions on the item parameters and latent variable means and (co)variances. Although the section header is required, there are no mandatory commands that must be included. Constraints and prior distributions may be placed on almost any aspect of the model, including means, covariances, and the various item parameters. Details on how to specify constraints will be discussed in examples that utilize constraints.

---

## 2.2. Single-Group 2PL Model Calibration

The first several examples will use data from two test forms that resemble those used in North Carolina end-of-grade testing. Data are responses to 12 multiple choice items collected from 2844 test-takers, with responses coded as correct (1) or incorrect (0). These dichotomous responses may be fitted with either the 2PL model, if guessing is assumed to be negligible, or with the 3PL model, which explicitly models guessing with an additional parameter. Example syntax for fitting the 2PL model is presented first.

### Example 2.1: 2PL Calibration Syntax

```
<Project>
Title = "2PL example";
Description = "12 items, 1 Factor, 1 Group 2PL Calibration";
<Options >
Mode = Calibration;
<Groups>
%Group1%
File = "g341-19.dat";
Varnames = v1,v2,v3,v4,v5,v6,v7,v8,v9,v10,v11,v12;
N = 2844;
Ncats(v1-v12) = 2;
Model(v1-v12) = Graded(2);
<Constraints>
```

Following the previous discussion, section headers (denoted by their enclosure within `<` and `>`) precede the commands. The `<Project>` header is followed by a title and description, both of which will be printed in the output. The `<Options>` section, by the `Mode` command, declares this to be a calibration run. After the `<Groups>` section header, a name for the first (and only) group is provided, which is enclosed within `%`s. Then the name of the data file (`File =`

" "); variable names (**Varnames** = ); number of examinees (**N** = ); number of response categories in the data for each item (**Ncats**( ) = ); and the IRT model to be fit to each item (**Model**( ) = ) are provided. In this example, only the file name was provided in the **File** statement. When the command file (\*.flexmirt) and the data file are in the same folder, the name of the data file is sufficient for flexMIRT® to locate it. If, however, the command file and the data are in different directories, it is necessary to specify the full path in the **File** statement. For example, an alternate statement might be **File** = "C:\Users\VPG\Documents\flexMIRT\example 2-1\g341-19.dat";.

When providing names for the variables in the **Varnames** statement, the name for every variable was fully written out as **v1**, **v2**, ..., **v12**. For the **Ncats** and **Model** statements, a shortcut was used, which is only available when variables names end with consecutive integers. As exemplified in the syntax, **v1**, **v2**, ..., **v12** can be shortened to **v1-v12**. This shorthand may be employed in any statement where variable names are provided.

For the **Model** statement, the Graded Response Model, with two categories, is specified as **Graded**(2). This model is formally equivalent to the 2PL model and as such flexMIRT® does not implement the 2PL model separately. Although not used in this example, it is useful to be aware that multiple **Ncat** and **Model** statements can be specified within one group, which is needed for an analysis of mixed item types. For example, if items 1 through 6 in the current data were dichotomous and items 7 through 12 were 5-category polytomous items, two **Ncat** statements (i.e., **Ncats**(**v1-v6**) = 2; **Ncats**(**v7-v12**) = 5;) and two **Model** statements (i.e., **Model**(**v1-v6**) = **Graded**(2); **Model**(**v7-v12**) = **Graded**(5);) would be provided to fit appropriate models to the items.

There are no constraints placed on this model, so after the section header is declared, the command file syntax ends. At the completion of estimation, flexMIRT® automatically opens the results in the Output viewer. The output is also saved into a text file named with the command file name and "-irt.txt" as the extension. The command file used in the first example is "2PLM\_example.flexmirt" and the output is saved to "2PLM\_example-irt.txt". The output is presented in a straight-forward fashion, so those familiar with IRT and its parameters and fit values should find it easily understandable. For completeness, however, we will review the output and highlight relevant sections. Due to the length of the output file, it will be separated into parts which are discussed in turn.

## Output 2.1: 2PL Calibration Output - Summary and Controls

```
flexMIRT(R) 2.00 (64-bit)
Flexible Multilevel Multidimensional Item Analysis and Test Scoring
(c) 2012-2013 Vector Psychometric Group, LLC., Chapel Hill, NC, USA

2PLM example
12 items 1 Factor, 1 Group 2PLM Calibration

Summary of the Data and Dimensions
  Missing data code      -9
  Number of Items       12
  Number of Cases      2844
  # Latent Dimensions    1

Item  Categories      Model
  1         2        Graded
  2         2        Graded
  3         2        Graded
  4         2        Graded
  5         2        Graded
  6         2        Graded
  7         2        Graded
  8         2        Graded
  9         2        Graded
 10         2        Graded
 11         2        Graded
 12         2        Graded

Bock-Aitkin EM Algorithm Control Values
Maximum number of cycles:   500
Convergence criterion:   1.00e-004
Maximum number of M-step iterations:   100
Convergence criterion for iterative M-steps:   1.00e-007
Number of rectangular quadrature points:   49
Minimum, Maximum quadrature points:   -6.00,   6.00
Standard error computation algorithm: Cross-product approximation

Miscellaneous Control Values
Z tolerance, max. abs. logit value:   50.00
```

```

Number of free parameters:    24
Number of cycles completed:   27
Number of processor cores used: 1
Maximum parameter change (P#): 0.000083709 ( 13)

Processing times (in seconds)
E-step computations:         0.23
M-step computations:         0.02
Standard error computations:  0.22
Goodness-of-fit statistics:   0.00
Total:                       0.47

Output Files
Text results and control parameters: 2PLM_example-irt.txt

Convergence and Numerical Stability
flexMIRT(R) engine status: Normal termination
First-order test: Convergence criteria satisfied
Condition number of information matrix: 46.0221
Second-order test: Solution is a possible local maximum

```

Following the flexMIRT<sup>®</sup> version and copyright information, the output begins by printing the title and description provided in the <Project> section of the syntax file. A broad summary of the data and model is provided on the next 4 lines, listing the missing value code, number of items, sample size, and total number of dimensions. In the next section, the number of categories and the IRT models are listed for each item. The various control values are listed (e.g., convergence criteria, the maximum number of iterations, number of quadrature points used, number of free parameters in the model) – if any control values were changed from defaults, the information printed here serves as verification.

Next, the program processing time is listed, both in total and broken down into various stages. Any output files that were generated by the analysis are named in the next section. The final part of this subsection of the output reports if flexMIRT<sup>®</sup> terminated normally and if the specified convergence criteria were met. The next section of the output provides the estimated item and group parameters.

## Output 2.2: 2PL Calibration Output - Item and Group Parameters

2PLM example									
12 items, 1 Factor, 1 Group 2PL Calibration									
2PL Items for Group 1: Group1									
Item	Label	P#	a	s.e.	P#	c	s.e.	b	s.e.
1	v1	2	1.05	0.07	1	0.30	0.05	-0.29	0.05
2	v2	4	1.23	0.11	3	2.88	0.11	-2.35	0.15
3	v3	6	0.84	0.06	5	-0.05	0.04	0.06	0.05
4	v4	8	1.04	0.07	7	0.94	0.05	-0.90	0.06
5	v5	10	0.85	0.06	9	0.33	0.04	-0.39	0.06
6	v6	12	1.34	0.10	11	2.09	0.09	-1.56	0.09
7	v7	14	1.90	0.17	13	4.29	0.21	-2.26	0.12
8	v8	16	0.97	0.08	15	1.51	0.06	-1.56	0.10
9	v9	18	1.89	0.15	17	3.35	0.16	-1.78	0.08
10	v10	20	1.32	0.10	19	2.40	0.09	-1.82	0.10
11	v11	22	0.87	0.06	21	-0.26	0.04	0.30	0.05
12	v12	24	1.01	0.07	23	0.15	0.05	-0.15	0.05
2PLM example									
12 items, 1 Factor, 1 Group 2PL Calibration									
Group Parameter Estimates:									
Group	Label	P#	mu	s.e.	P#	s2	s.e.	sd	s.e.
1	Group1		0.00	----		1.00	----	1.00	----

Following a reprinting of the title and description, each item is listed in its own row, with the provided label and assigned parameter numbers, estimated item parameters, and standard error values going across. The IRT model fit in this example was the 2PLM, so the reported item parameters are the slope (labeled a) and intercept (labeled c). In addition, for unidimensional models the difficulty value (labeled b) is also reported; this value may be found from the other two parameters as  $b = c / -a$ . Below the item parameter table, the group parameters are printed. In this example, we had only one group, so the mean and variance were fixed at 0 and 1, respectively, for model identification. That these parameters were not estimated is indicated by both the lack of an assigned parameter number and the dashes in the standard error columns.

### Output 2.3: 2PL Calibration Output - Information Functions

2PLM example																
12 items, 1 Factor, 1 Group 2PL Calibration																
Item Information Function Values at 15 Values of theta from -2.80 to 2.80 for Group 1: Group1																
Theta:																
Item	Label	-2.8	-2.4	-2.0	-1.6	-1.2	-0.8	-0.4	-0.0	0.4	0.8	1.2	1.6	2.0	2.4	2.8
1	v1	0.07	0.10	0.13	0.18	0.22	0.26	0.28	0.27	0.24	0.20	0.16	0.12	0.08	0.06	0.04
2	v2	0.35	0.38	0.36	0.31	0.24	0.17	0.12	0.08	0.05	0.03	0.02	0.01	0.01	0.00	0.00
3	v3	0.05	0.07	0.09	0.11	0.13	0.15	0.17	0.18	0.17	0.16	0.14	0.12	0.10	0.08	0.06
4	v4	0.12	0.16	0.20	0.24	0.26	0.27	0.25	0.22	0.18	0.13	0.10	0.07	0.05	0.03	0.02
5	v5	0.07	0.09	0.12	0.14	0.16	0.17	0.18	0.17	0.16	0.14	0.12	0.09	0.07	0.06	0.04
6	v6	0.24	0.33	0.41	0.45	0.42	0.35	0.26	0.18	0.11	0.07	0.04	0.03	0.02	0.01	0.01
7	v7	0.70	0.89	0.85	0.62	0.38	0.20	0.10	0.05	0.02	0.01	0.01	0.00	0.00	0.00	0.00
8	v8	0.17	0.20	0.22	0.24	0.23	0.21	0.17	0.14	0.11	0.08	0.06	0.04	0.03	0.02	0.01
9	v9	0.39	0.64	0.85	0.87	0.67	0.42	0.23	0.12	0.06	0.03	0.01	0.01	0.00	0.00	0.00
10	v10	0.29	0.38	0.43	0.43	0.37	0.29	0.20	0.13	0.08	0.05	0.03	0.02	0.01	0.01	0.00
11	v11	0.04	0.06	0.08	0.10	0.13	0.15	0.17	0.19	0.19	0.18	0.16	0.14	0.11	0.09	0.07
12	v12	0.06	0.09	0.12	0.16	0.20	0.23	0.25	0.25	0.24	0.20	0.17	0.13	0.09	0.07	0.05
Test Information:		3.56	4.38	4.86	4.83	4.41	3.87	3.38	2.97	2.61	2.29	2.01	1.77	1.58	1.42	1.30
Expected s.e.:		0.53	0.48	0.45	0.45	0.48	0.51	0.54	0.58	0.62	0.66	0.71	0.75	0.80	0.84	0.88
Marginal Reliability for Response Pattern Scores: 0.64																

The item and test information functions, values for which are presented in Output 2.3, are the focus of the next section of the output file. The title and description are printed once again, followed by the label for the information table. As may be seen, the values of theta used in the table range from -2.8 to 2.8, increasing in steps of 0.4. The information function values for each item are presented on the appropriate row and the Test Information Function (TIF) values, which sum the item information values at each theta value (plus the contribution from the prior), are printed towards the bottom of the table. Additionally, the expected standard error associated with each theta value is printed. Finally, the marginal reliability, which is defined as 1 minus the averaged error variance divided by the prior variance, is reported underneath the table. After the information and reliability section, the overall goodness of fit (GOF) indices are presented, which appear in the next output box.

**Output 2.4:** 2PLM Calibration Output - Goodness of Fit Indices

Statistics based on the loglikelihood of the fitted model		
	-2loglikelihood:	33408.05
Akaike Information Criterion (AIC):		33456.05
Bayesian Information Criterion (BIC):		33598.92
Full-information fit statistics of the fitted model		
The table is too sparse to compute the general		
multinomial goodness of fit statistics.		
Limited-information fit statistics of the fitted model		
The M2 statistics were not requested.		

When no option is specified, the GOF reporting will provide only basic fit indices. These include the values for the  $-2 \times \text{Log Likelihood}$ , the Akaike Information Criterion (AIC), Bayesian Information Criterion (BIC), and when appropriate, the Pearson  $X^2$  statistic (labeled **X2**), the likelihood ratio statistic (labeled **G2**), their associated degrees of freedom, the estimated population discrepancy function value (labeled **F0hat**), and the RMSEA (Root Mean Square Error of Approximation; e.g., Browne & Cudeck, 1993). In this example, flexMIRT<sup>®</sup> has reported that the underlying contingency table was too sparse to compute the full-information fit values because the total number of cross-classifications is  $2^{12} = 4096$ , which is considerably larger than the sample

size of 2844.

## 2.3. Single-Group 3PL Model Calibration

The data in the previous section may also be suitable for the 3PL model, which explicitly models examinees' guessing.

### Example 2.2: Single Group 3PL Calibration Syntax

```
<Project>
Title = "3PL example";
Description = "12 items, 1 Factor, 1 Group Calibration,
saving parameter estimates";
<Options>
Mode = Calibration;
GOF = Extended;
savePRM = Yes;
<Groups>
%Group1%
File = "g341-19.dat";
Varnames = v1-v12;
N = 2844;
Ncats(v1-v12) = 2;
Model(v1-v12) = ThreePL;
<Constraints>
Prior (v1-v12), Guessing : Beta(1.0,4.0);
```

The syntax for this run differs from the previous one in several ways. In the `<Options>` section, the parameter estimates are to be saved to a separate output file (`savePRM = Yes;`) and additional GOF statistics were requested by specifying `GOF = Extended;` as an option. In the `<Groups>` section the fitted model is now the 3PL model (`Model` keyword is `ThreePL`). In the `<Constraints>` section, a prior distribution is imposed on the **Guessing** parameters for all items. In this example the `Beta(1.0,4.0)` distribution was used, which corresponds to a prior sample size of 5, with  $1.0/(1.0 + 4.0) = 0.2$  as its mode. This prior indicates that the prior guessing probability is equal to 0.2, as these multiple-choice items have 5 options. The `Normal` and `LogNormal` distributions are also available when specifying priors. In relation to the lower asymptote, the `Normal` prior applies a normal distribution prior on the *logit* of the lower asymptote parameter. A prior of `Normal(-1.39, 0.5)` is a reason-

able prior for the lower asymptote, with a mode around 0.20 in the typical  $g$  metric. Other common normal priors for the logit- $g$  parameter would be  $N(-1.09, 0.5)$  for items with 4 possible response options,  $N(-0.69, 0.5)$  for items with 3 response options, and  $N(0.0, 0.5)$  for True/False type items. The standard deviation of 0.5 is standard across all noted priors and is based primarily on experience, in that it provides a prior that is diffuse enough to allow estimated guessing parameters to move away from the mean value if necessary but not so diffuse as to be uninformative.

The output for this analysis is quite similar to that of the previous run, so only differences will be highlighted. As noted in the syntax discussion, flexMIRT<sup>®</sup> has been instructed to save the item and group parameter values into a separate file. This results in an additional file name, with the “\*-prm.txt” extension, being listed in the output files. The layout of the item parameter output table (presented in Output 2.5) is essentially the same as before, with the slopes, thresholds, and difficulty values and their standard errors reported first. In the last four columns of the table, the estimates for the lower asymptote (both in its logit form and in terms of the pseudo-guessing probability) are reported, as well as standard errors.

**Output 2.5:** Single Group 3PL Calibration Output- Item Parameters

```

...
Output Files
Text results and control parameters: g341-19-irt.txt
Text parameter estimate file: g341-19-prm.txt
...
3PL example
12 items, 1 Factor, 1 Group Calibration, saving parameter estimates

3PL Items for Group 1: Group1

```

Item	Label	P#	a	s.e.	P#	c	s.e.	b	s.e.	P#	logit-g	s.e.	g	s.e.
1	v1	3	1.84	0.30	2	-0.56	0.26	0.31	0.10	1	-1.09	0.22	0.25	0.04
2	v2	6	1.27	0.12	5	2.77	0.20	-2.17	0.26	4	-2.00	1.33	0.12	0.14
3	v3	9	1.50	0.26	8	-0.98	0.29	0.65	0.11	7	-1.19	0.23	0.23	0.04
4	v4	12	1.31	0.19	11	0.46	0.22	-0.35	0.21	10	-1.19	0.45	0.23	0.08
5	v5	15	1.35	0.22	14	-0.42	0.25	0.31	0.15	13	-1.08	0.28	0.25	0.05
6	v6	18	1.39	0.15	17	1.91	0.18	-1.37	0.23	16	-1.93	1.09	0.13	0.12
7	v7	21	1.96	0.18	20	4.27	0.25	-2.18	0.17	19	-2.22	1.46	0.10	0.13
8	v8	24	1.05	0.13	23	1.24	0.23	-1.19	0.34	22	-1.56	0.91	0.17	0.13
9	v9	27	1.94	0.18	26	3.31	0.19	-1.71	0.15	25	-2.59	1.59	0.07	0.10
10	v10	30	1.35	0.13	29	2.28	0.17	-1.69	0.22	28	-2.23	1.36	0.10	0.12
11	v11	33	1.77	0.32	32	-1.40	0.34	0.79	0.08	31	-1.25	0.19	0.22	0.03
12	v12	36	1.55	0.24	35	-0.52	0.23	0.33	0.11	34	-1.37	0.28	0.20	0.05

In addition to the added parameters and output files, the **GOF** statement was set to **Extended**, so additional fit statistics, namely marginal  $X^2$  values and the local dependence (LD) statistics of Chen and Thissen (1997) are included in the output. A table containing these values is printed between the group parameter values and the Information Function values table. This table is presented below. The values in the second column of this table are the item-specific marginal  $X^2$ , which may be tested against a chi-square variable with degrees of freedom equal to the number of categories for that item minus 1. In this case, all items fit well at the level of univariate margins. The remaining cells provide pairwise diagnostic information for possible local dependence. These are standardized  $X^2$  values so values larger than 3 in absolute magnitude should be examined further.

**Output 2.6:** Single Group 3PL Calibration Output- Extended GOF Table

Marginal fit (Chi-square) and Standardized LD X2 Statistics for Group 1: Group1											
Marginal											
Item	Chi2	1	2	3	4	5	6	7	8	9	10
1	0.0										
2	0.0	-0.7									
3	0.0	-0.7	2.0								
4	0.0	0.9	-0.6	0.2							
5	0.0	-0.6	-0.0	-0.1	5.3						
6	0.0	-0.5	-0.7	-0.7	-0.5	-0.3					
7	0.0	0.8	-0.4	-0.4	-0.6	0.3	-0.7				
8	0.0	-0.5	-0.6	-0.7	0.6	-0.4	0.7	0.0			
9	0.0	-0.6	0.7	-0.4	-0.4	-0.5	-0.7	4.4	0.2		
10	0.0	2.5	5.0	-0.7	0.1	-0.5	-0.0	-0.0	-0.7	1.4	
11	0.0	-0.6	3.5	-0.1	-0.3	-0.4	0.8	0.0	-0.7	-0.7	-0.3
12	0.0	-0.7	-0.6	3.7	0.3	-0.0	-0.3	-0.7	-0.6	0.4	-0.1
Marginal											
Item	Chi2	11									
11	0.0										
12	0.0	1.2									

## 2.4. Single-Group 3PL EAP Scoring

There are situations when existing item parameters will be used and only scoring for a new set of data is desired. This next example will introduce a command file that does just that, using the saved parameter estimates from a prior calibration run.

### Example 2.3: 3PL EAP Scoring example

```
<Project>
Title = "3PL EAP scoring";
Description = "12 items, 1 Factor, 1 Group Scoring";
<Options>
Mode = Scoring;
ReadPRMFile = "3PL_example-prm.txt";
Score = EAP;
saveSCO=YES;
<Groups>
%Group1%
File = "g341-19.dat";
Varnames = v1-v12;
N =2844;
<Constraints>
```

For scoring runs the <Project> section stays the same. The first change encountered is in the <Options> section, in that **Mode** is now set to **Scoring**. Using the command **ReadPRMFile**, the name of the file that contains the item parameter estimates is provided. In this case, the file was created by the previous calibration run. The type of IRT scale scores is selected using the **Score =** ; command. In this case, Expected *A Posteriori* (EAP) scores will be saved. We may also specify **ML** for maximum likelihood scoring, **MAP** for Maximum *A Posteriori* scoring, or **SSC** for summed score to EAP conversion tables. We have also specified that the scores should be saved into a separate file, which will be labeled with the extension “\*-sco.txt”. The command requesting flexMIRT<sup>®</sup> to output the estimated scores (**SaveSco = Yes;**) is redundant for most scoring methods, but it is needed if individual scores are desired for the **SSC** method. Without the **SaveSco = Yes;** command, **SSC** will produce only the summed score to scale score conversion table. Notice that this may be a desirable feature in some situations, when only the conversion table is needed with actual scoring occurring at a much later stage.

The output of scoring runs differs from calibration runs so we have reproduced the entire scoring control output file, which has the extension “\*-ssc.txt”, in Output 2.7 below.

## Output 2.7: 3PL EAP Scoring Output

3PL EAP scoring				
12 items, 1 Factor, 1 Group Scoring				
Summary of the Data and Control Parameters				
Missing data code		-9		
Number of Items		12		
Number of Cases		2844		
Number of Dimensions		1		
Item	Categories		Model	
1	2		3PL	
2	2		3PL	
3	2		3PL	
4	2		3PL	
5	2		3PL	
6	2		3PL	
7	2		3PL	
8	2		3PL	
9	2		3PL	
10	2		3PL	
11	2		3PL	
12	2		3PL	
Scoring Control Values				
Response pattern EAPs are computed				
Miscellaneous Control Values				
Output Files				
Text results and control parameters: g341-19-ssc.txt				
Text scale score file: g341-19-sco.txt				
3PL EAP scoring				
12 items, 1 Factor, 1 Group Scoring				
3PL Items for Group 1: Group1				
Item	a	c	b logit-g	g

1	1.84	-0.56	0.31	-1.09	0.25
2	1.27	2.77	-2.17	-2.00	0.12
3	1.50	-0.98	0.65	-1.19	0.23
4	1.31	0.46	-0.35	-1.19	0.23
5	1.35	-0.42	0.31	-1.08	0.25
6	1.39	1.91	-1.37	-1.93	0.13
7	1.96	4.27	-2.18	-2.22	0.10
8	1.05	1.24	-1.19	-1.56	0.17
9	1.94	3.31	-1.71	-2.59	0.07
10	1.35	2.28	-1.69	-2.23	0.10
11	1.77	-1.40	0.79	-1.25	0.22
12	1.55	-0.52	0.33	-1.37	0.20
3PL EAP scoring					
12 items, 1 Factor, 1 Group Scoring					
Group Parameter Estimates:					
Group	Label	mu	s2	sd	
1	Group1	0.00	1.00	1.00	

As with the calibration output, the file starts with a general data summary and echoing of values that were provided to flexMIRT®. In the scoring mode, this section of the output is much shorter due to the fact that fewer control parameters are available for modification. Of note in this section is the line labeled “Scoring Control Values,” under which the type of IRT scale scores is printed. Below that, the output files generated are named and tables containing the item and group parameters, as read in from the parameter file are printed, to allow for the user to verify the parameter values were read in as expected.

The individual scores were saved into a separate file with the extension “\*-sco.txt.” Opening the individual score file (3PLM\_score\_example-sco.txt) reveals that the columns in the score file are formatted as follows: the group number, the observation number, the scale score, and the standard error associated with the scale score. Optionally, following the group number and the observation number, flexMIRT® will print the case ID number if the `caseID` variable is given in the command syntax.

---

## 2.5. Single-Group 3PL ML Scoring

As noted, maximum likelihood scores are also available. One issue that may occur when using ML scores is that extreme response patterns on either end of the continuum (e.g., all correct or all incorrect) are associated with undefined or unreasonable theta estimates. When **Score = ML**; the user is required to specify the desired maximum and minimum scores via the **MaxMLscore** and **MinMLscore** keywords, to obtain practical theta estimates for such cases.

### Example 2.4: 3PL ML Scoring example

```
<Project>
Title = "3PL ML Scoring Example";
Description = "12 items 1 Factor, 1 Group Scoring";

<Options>
Mode = scoring;
ReadPRMFile = "3PL_example-prm.txt";
Score= ML;
MaxMLscore = 3.5;
MinMLscore = -5.5;
saveSCO = YES;

<Groups>
%Group1%
File = "g341-19.dat";
Varnames = v1-v12;
N = 2844;

<Constraints>
```

This syntax is the same as the previous scoring example, save for the change from **Score = EAP** to **Score = ML** and the additional commands **MaxMLscore** and **MinMLscore** used to “reign in” the extreme ML estimates. We present the first 10 cases of the associated -sco.txt file, where the scores were saved.

### Output 2.8: 3PL ML Scoring Output

1	1	2	-0.198234	0.573388
1	2	3	1.427265	0.784667
1	3	2	0.113716	0.680658
1	4	3	1.427265	0.784667
1	5	3	-0.675141	0.625639
1	6	3	0.637464	0.606990
1	7	3	1.382694	0.792826
1	8	3	-1.205723	0.628273
1	9	2	3.500000	99.990000
1	10	3	0.859248	0.658306

Like the previous -sco file, the first column indicates group membership and the second column provides the flexMIRT<sup>®</sup> assigned observation number. The third column reports the number of iterations to reach the reported score. The fourth and fifth columns are the ML theta estimate and the associated SE, respectively. As can be seen for observation 9, our specified maximum of 3.5 was applied. Because the reported value was assigned as a result of our **MaxMLscore** statement, the reported SE is 99.99 to indicate that the score was assigned, rather than estimated.

## 2.6. Graded Model Calibration and Scoring

Example syntax for a combined calibration and scoring run is now presented. flexMIRT<sup>®</sup> code for the graded response model, which may be used with two or more ordered response options (e.g., Samejima, 1969), is also introduced. This example employs data from 3000 simulees, using parameter values reported in Table 1 of Edwards' (2009) paper as generating values. These data were constructed to mimic responses collected on the Need for Cognition Scale (Cacioppo, Petty, & Kao, 1984), which is an 18 item scale that elicits responses on a 5-point Likert-type scale, asking participants to indicate the extent to which the content of the item prompt describes them.

**Example 2.5:** Graded Model Combined Calibration and Scoring Syntax with Recoding

```
<Project>
Title = "NCS example";
Description = "18 items, 1D, 1 Group Calibration and Scoring";
```

```

<Options>
Mode = Calibration;
SaveSCO = YES;
Score = MAP;
GOF = Extended;
<Groups>
%Group1%
File = "NCSsim.dat";
Varnames = v1-v18;
N = 3000;
Ncats(v1-v18) = 5;
Code(v1-v18) = (1,2,3,4,5),(0,1,2,3,4);
Model(v1-v18) = Graded(5);
<Constraints>

```

The only new command encountered in this example is the "Code =" statement in the <Groups> section. The internal representation of item response data in flexMIRT<sup>®</sup> is zero-based (i.e., response options must start at zero and go up). The raw item response data file for this example (NC-Ssim.dat) labels the categories as 1 through 5, so recoding is necessary. The Code statement does this for variables v1-v18 by specifying the original values in the first pair of parentheses and, following a comma, the desired recoded values in a second set of parentheses. Thus, a command such as Code(v1-v18)=(1,2,3,4,5),(4,3,2,1,0); accomplishes both reverse coding and makes the recoded item response data zero-based.

Another point of note in this command file is that, even though this is a calibration run, we have requested MAP scores be saved. This combination of commands is a short-cut to a combined calibration and scoring run, in which the calibration is explicitly requested and the SaveSco=Yes; command implicitly requests a scoring run, using the estimated parameter values from the calibration.

With respect to the output, the primary change from previous examples are the additional parameter values in the item parameter table. As may be seen from the two item parameter estimate output tables below there are multiple thresholds (or equivalently, intercepts) for each item. For convenience, both of these parameterizations (thresholds and intercepts) are presented in separate tables. However, only the intercept values will be saved to the -prm file, if it is requested. This behavior can be modified by adding the statement SlopeThreshold= Yes; to the <Options> section. When the

SlopeThreshold keyword is invoked threshold values, rather than intercept values, will be saved to the -prm file.

**Output 2.9:** Graded Model Calibration - Slopes and Intercepts

Graded Items for Group 1: Group1																
Item	Label	P#	a	s.e.	P#	c 1	s.e.	P#	c 2	s.e.	P#	c 3	s.e.	P#	c 4	s.e.
1	v1	5	1.58	0.06	1	3.25	0.09	2	0.95	0.06	3	-0.41	0.05	4	-3.34	0.09
2	v2	10	1.85	0.06	6	4.35	0.12	7	1.79	0.07	8	0.48	0.06	9	-2.91	0.09
3	v3	15	1.79	0.06	11	4.04	0.11	12	1.61	0.06	13	0.29	0.06	14	-2.40	0.08
4	v4	20	1.73	0.06	16	4.11	0.12	17	1.65	0.06	18	0.35	0.06	19	-2.61	0.08
5	v5	25	1.44	0.05	21	4.35	0.12	22	1.71	0.06	23	0.47	0.05	24	-2.35	0.07
6	v6	30	1.19	0.05	26	2.10	0.06	27	0.29	0.05	28	-0.88	0.05	29	-3.30	0.09
7	v7	35	1.24	0.05	31	3.27	0.09	32	0.77	0.05	33	-0.04	0.05	34	-2.61	0.07
8	v8	40	0.72	0.04	36	2.18	0.06	37	0.02	0.04	38	-0.97	0.04	39	-2.95	0.08
9	v9	45	1.08	0.05	41	2.46	0.07	42	-0.10	0.05	43	-1.06	0.05	44	-3.06	0.08
10	v10	50	1.37	0.05	46	4.72	0.15	47	2.28	0.07	48	0.78	0.05	49	-2.12	0.07
11	v11	55	1.62	0.06	51	4.34	0.12	52	1.92	0.07	53	0.72	0.06	54	-2.62	0.08
12	v12	60	1.44	0.05	56	4.41	0.13	57	1.79	0.06	58	0.75	0.05	59	-2.03	0.07
13	v13	65	1.12	0.05	61	2.19	0.06	62	0.35	0.05	63	-1.09	0.05	64	-3.72	0.10
14	v14	70	1.37	0.05	66	3.71	0.10	67	1.71	0.06	68	0.42	0.05	69	-2.01	0.07
15	v15	75	1.57	0.06	71	4.30	0.12	72	1.66	0.06	73	0.30	0.05	74	-2.47	0.07
16	v16	80	0.80	0.04	76	2.13	0.06	77	0.20	0.04	78	-0.59	0.04	79	-2.69	0.07
17	v17	85	1.17	0.05	81	3.28	0.09	82	0.95	0.05	83	0.00	0.05	84	-2.07	0.06
18	v18	90	0.61	0.04	86	2.83	0.08	87	1.16	0.05	88	0.21	0.04	89	-2.13	0.06

**Output 2.10:** Graded Model Calibration - Slopes and Thresholds

Graded Items for Group 1: Group1												
Item	Label	P#	a	s.e.	b 1	s.e.	b 2	s.e.	b 3	s.e.	b 4	s.e.
1	v1	5	1.58	0.06	-2.05	0.07	-0.60	0.04	0.26	0.03	2.11	0.07
2	v2	10	1.85	0.06	-2.35	0.08	-0.97	0.04	-0.26	0.03	1.58	0.05
3	v3	15	1.79	0.06	-2.26	0.07	-0.90	0.04	-0.16	0.03	1.34	0.05
4	v4	20	1.73	0.06	-2.38	0.08	-0.95	0.04	-0.21	0.03	1.51	0.05
5	v5	25	1.44	0.05	-3.01	0.12	-1.19	0.05	-0.33	0.04	1.63	0.06
6	v6	30	1.19	0.05	-1.77	0.07	-0.24	0.04	0.74	0.05	2.77	0.11
7	v7	35	1.24	0.05	-2.63	0.10	-0.62	0.04	0.03	0.04	2.10	0.08
8	v8	40	0.72	0.04	-3.05	0.17	-0.03	0.06	1.36	0.09	4.12	0.23
9	v9	45	1.08	0.05	-2.29	0.10	0.09	0.04	0.99	0.05	2.84	0.12
10	v10	50	1.37	0.05	-3.45	0.14	-1.67	0.06	-0.57	0.04	1.56	0.06
11	v11	55	1.62	0.06	-2.68	0.09	-1.19	0.05	-0.45	0.03	1.62	0.06
12	v12	60	1.44	0.05	-3.05	0.11	-1.24	0.05	-0.52	0.04	1.40	0.05
13	v13	65	1.12	0.05	-1.95	0.08	-0.31	0.04	0.97	0.05	3.31	0.14
14	v14	70	1.37	0.05	-2.7	0.10	-1.25	0.05	-0.30	0.04	1.47	0.06
15	v15	75	1.57	0.06	-2.74	0.10	-1.06	0.04	-0.19	0.03	1.57	0.06
16	v16	80	0.80	0.04	-2.67	0.14	-0.25	0.05	0.75	0.06	3.38	0.18
17	v17	85	1.17	0.05	-2.8	0.11	-0.81	0.05	0.00	0.04	1.77	0.07
18	v18	90	0.61	0.04	-4.63	0.31	-1.89	0.13	-0.35	0.07	3.48	0.23

As before, the marginal  $X^2$  values and the local dependence (LD) statistic values of Chen and Thissen (1997) are reported due to the GOF = Extended;

statement, but an additional feature is present in the table because of the ordinal nature of the data. The **Marginal fit (Chi-square) and Standardized LD X2 Statistics** table is presented in the following Output example. The standardized LD  $X^2$  value for each item pair is followed by either the letter “p” or the letter “n”. When the item responses are composed of ordered categories, phi correlations are computed for both the model-implied and observed bivariate tables. If the model implied correlation is lower than the observed correlation for a given item pair, a “p” is printed after the calculated  $X^2$ , meaning “positive” LD. If the model implied correlation is higher, an “n” is printed, indicating “negative” LD. Either a “p” or an “n” is printed after every value in the table. The absolute magnitude of the individual  $X^2$  values should still be evaluated against a critical value, such as 3.0, to determine if a non-ignorable level of LD is present.

**Output 2.11:** Graded Model Calibration - Extended GOF Table

Marginal fit (Chi-square) and Standardized LD X2 Statistics for Group 1: Group1											
Item	Marginal Chi2	1	2	3	4	5	6	7	8	9	10
1	0.0										
2	0.1	-0.7n									
3	0.0	-0.8p	-0.3n								
4	0.1	-0.6n	-0.7p	0.7n							
5	0.1	1.0n	-1.4p	-0.6p	-0.8n						
6	0.1	-0.2n	-1.5n	-0.9p	-0.3n	0.7p					
7	0.0	-0.4p	-1.4p	-0.8p	0.0n	-0.2n	0.1p				
8	0.0	-1.2n	-0.1n	-1.1n	-0.1n	-0.3p	-1.5p	-1.7p			
9	0.0	0.6n	0.6p	-0.5p	0.3p	1.4n	-0.9p	-0.8p	-1.5p		
10	0.0	-0.4p	-1.3p	1.0n	-0.7n	-0.2p	-0.5p	-1.7n	-1.2p	-0.7n	
11	0.1	-0.0p	-0.9n	1.3p	-1.2p	-0.2p	0.7n	-1.1p	0.1n	-0.4p	0.4p
12	0.0	-0.3p	0.9n	-1.1p	0.6p	0.8p	-0.1n	-0.7n	-0.5n	-0.2n	-1.9p
13	0.1	-1.1p	-0.9n	0.5p	0.4n	-0.4n	-0.7n	1.4p	-1.5p	-0.3p	-1.1n
14	0.1	-0.4p	-1.1n	-0.7n	0.8n	-0.7n	0.8p	0.7p	0.3n	0.7n	0.5p
15	0.0	-1.3n	0.0p	-1.0p	-0.6n	0.4p	-1.2p	-0.9n	-0.4n	-0.5p	-0.6p
16	0.0	-0.5p	1.2n	-0.6p	-1.3n	-0.4p	1.4p	0.1p	-0.2p	-0.1n	0.4p
17	0.0	-0.8n	-0.8p	0.6p	-1.4p	-0.8p	0.7n	-0.7n	-0.1p	-0.9n	-0.6p
18	0.0	1.8p	0.9p	-0.6n	-0.9p	2.7n	1.5p	0.5n	-0.4n	-0.3p	-0.2p
Item	Marginal Chi2	11	12	13	14	15	16	17			
11	0.1										
12	0.0	0.5p									
13	0.1	-0.6p	-1.0p								
14	0.1	-1.1p	-0.9n	-1.8p							
15	0.0	-1.3n	1.0n	0.5p	-0.4n						
16	0.0	2.6n	-1.9p	-1.2n	-0.5p	-1.5n					
17	0.0	1.8n	-1.7p	0.1n	-0.9n	-0.3p	0.7p				
18	0.0	-0.5p	-1.0p	-0.7n	0.6n	-0.3n	-0.1n	0.7p			

## 2.7. 1PL Model Fitted to Response Pattern Data

Up to this point, analyses have been conducted on raw (individual) data, in which each line in the data set represents responses from an individual. flexMIRT<sup>®</sup> is also able to handle data that are grouped by response patterns.

**Example 2.6:** 1PL Calibration and Scoring with Response Pattern Data

```
<Project>
  Title = "LSAT 6";
  Description = "5 Items 1PL N=1000 Grouped Data";
<Options>
  Mode = Calibration;
  GOF = Complete;
  JSI = Yes;
  HabermanResTbl = Yes;
  SX2Tbl = Yes;
  FactorLoadings = Yes;
  SaveSco = No;
  Score = SSC;
<Groups>
%Group1%
  File = "lsat6grp.dat";
  Varnames = v1-v5,w;
  Select = v1-v5; // only the first 5 variables
  CaseWeight = w;
  N = 30;
  Ncats(v1-v5) = 2;
  Model(v1-v5) = Graded(2);
<Constraints>
  Equal (v1-v5), Slope;
/* Entire
  Block of
  Comments */
```

The syntax file above uses the well-known LSAT6 data set, in which the individuals have been grouped by response patterns. There are 1000 examinees and 5 dichotomous items. The first use of constraints in the **<Constraints>** section is also presented, demonstrating how to fit a 1PL model in flexMIRT®.

An examination of the data file (lsat6grp.dat) shows that that the data for these five items have been grouped by response patterns with the last variable in the file providing the number of observations per pattern. After naming the group and file name in the **<Groups>** section as usual, we use the **Select** statement to select only the first 5 variables that will be subject to

IRT analysis (i.e., we exclude the weighting variable, which was named “w” in the **Varnames** statement). Note that this is also the first time that we have encountered comments in the syntax file. A comment begins with 2 slashes, following the standard C/C++ convention. They may appear anywhere in the syntax, but not within a command statement. In addition, one may also use a pair of `/* */` to create entire blocks of comments that span more than one lines, as shown at the end of the syntax file.

Returning to the example syntax, we use the **CaseWeight** command to identify the variable that contains response pattern frequencies, (i.e., the number of cases per response pattern). The rest of the **<Groups>** statements, those defining number of categories, etc., are as before.

Note that the graded model with 2 categories is specified in the **Model** statement, which is equivalent to a 2PL model. To obtain a 1PL model, the slope/discrimination parameters must be constrained equal across items. To accomplish this, an equality constraint is specified in the **<Constraints>** section using the **Equal** statement on the **Slope** parameters of items **v1-v5**. The keywords for other items parameters are **Intercept** for the location parameter in all models, **Guessing** for the guessing parameter/lower asymptote in the 3PL model, and **ScoringFn** for the scoring function parameters in the Nominal Categories model.

There are several new commands in the **<Options>** section. For the goodness-of-fit indices, the full gamut of available **GOF** indices (detailed in Chapter 7 of the manual) are requested by the keyword **Complete**. The **JSI** statement requests the Jackknife Slope Index (JSI), an LD detection tool, for all pairs of items. The **HabermanResTbl** command requests a table that lists the observed and expected frequencies and standardized residuals for each response pattern, as well as the associated SEs, EAPs and SDs. When **SX2Tbl** is requested, detailed item fit tables will be printed, in addition to the Orlando and Thissen (2000)  $S - X^2$  item fit chi-square statistics. The command **FactorLoadings = Yes;** requests the printing of the factor loading values for each item, which may be converted from the respective slope values (e.g., Takane & de Leeuw, 1987; Wirth & Edwards, 2007). Even though the scores are not saved (**SaveSco = No;**), the scoring method of **SSC** is specified. As noted previously, the **SSC** method will print the requested summed score to EAP scale score conversion table in the output, although no individual scores are calculated/saved. The commands introduced in this example add several

new parts to the output file.

**Output 2.12:** 1PL Grouped Data Output - Item Parameters and Factor Loadings

LSAT 6									
5 Items 1PL N=1000 Grouped Data									
2PL Items for Group 1: Group1									
Item	Label	P#	a	s.e.	P#	c	s.e.	b	s.e.
1	v1	6	0.76	0.00	1	2.73	0.01	-3.61	0.01
2	v2	6	0.76	0.00	2	1.00	0.00	-1.32	0.00
3	v3	6	0.76	0.00	3	0.24	0.00	-0.32	0.00
4	v4	6	0.76	0.00	4	1.31	0.00	-1.73	0.00
5	v5	6	0.76	0.00	5	2.10	0.00	-2.78	0.01

LSAT 6									
5 Items 1PL N=1000 Grouped Data									
Factor Loadings for Group 1: Group1									
Item	Label	lambda	1	s.e.					
1	v1	0.41	0.00						
2	v2	0.41	0.00						
3	v3	0.41	0.00						
4	v4	0.41	0.00						
5	v5	0.41	0.00						

The item parameters are printed in their customary order, but note that the 1PL equal-slope constraint specified in the syntax has been interpreted correctly, as evidenced by all values in the *a* column being equal. The factor loadings are presented below the typical item parameter table, with loadings appearing in the *lambda1* column.

The next new portion of the output are the values and tables associated with the summed-score based item fit diagnostic (e.g., Orlando & Thissen, 2000), which are printed as a result of the *GOF = Complete* and *SX2Tbl = Yes*; commands, respectively. The  $S - X^2$  value and table for the first item is presented. When the *p*-value associated with the  $S - X^2$  value is greater than the  $\alpha$ -level (typically 0.05), the item should be examined for possible misfit.

**Output 2.13: 1PL Grouped Data Output - Item Fit Diagnostics**

Orlando-Thissen-Bjorner Summed-Score Based Item Diagnostic Tables and X2s					
Group 1: Group1					
Item 1 S-X2(1) = 48.5, p < 0.0001					
	Category 0		Category 1		
Score	Observed	Expected	Observed	Expected	
0	1	0.4		1	1.6
1	4	5 1.1	1.5	4	6.9
2	4	1.0		6	9.0
3	4	8 0.6	1.5	4	7.4
4		1	0.1	1	1.9

By specifying **Score = SSC**; in the command file, we requested a summed score to EAP scale score conversion table be printed, which is shown below. With a total of 5 items, there are 6 possible summed scores which are listed in the first column. Going across a row, after the sum score, the EAP score and the posterior standard deviation associated with that particular summed score value are presented. Additionally, the table also includes model-implied and observed probabilities for the summed scores, appearing in columns labeled P and O, respectively. Immediately after the table, the marginal reliability of the scale scores is printed, as well as the value of a chi-square statistic that provides a test of the underlying normality of the latent variable distribution.

**Output 2.14: 1PL Grouped Data Output - Summed Score to Scale Score Conversion**

Summed Score to Scale Score Conversion Table:				
Summed				
Score	EAP	SD	P	O
0.00	-1.910	0.797	0.0023654	0.0030000
1.00	-1.429	0.800	0.0208898	0.0200000
2.00	-0.941	0.809	0.0885974	0.0850000
3.00	-0.439	0.823	0.2274460	0.2370000
4.00	0.084	0.841	0.3648997	0.3570000
5.00	0.632	0.864	0.2958018	0.2980000
Summed score based latent distribution fit S-D2 = 0.9, p = 0.8166				
Marginal reliability of the scaled scores for summed scores = 0.29427				

Scrolling further down in the full output file, we encounter sections related to LD detection. We have previously discussed the Marginal fit and Standardized LD  $X^2$  Statistics table, but the matrix immediately below that in the

output is new. The `JSI = Yes;` statement in the command file produces a matrix (labeled Edwards-Houts-Cai Jackknife Slope Diagnostic Index) which provides values for a newly introduced LD detection method (Edwards & Cai, 2011), the calculation of which is described in detail later in the manual.

**Output 2.15:** 1PL Grouped Data Output - JSI matrix

Edwards-Houts-Cai Jackknife Slope Diagnostic Index for Group 1: Group1					
Item	1	2	3	4	5
1	---	0.3	0.3	0.3	0.3
2	-0.7	---	-0.7	-0.7	-0.7
3	2.5	2.5	---	2.5	2.5
4	-1.4	-1.4	-1.4	---	-1.4
5	-1.1	-1.1	-1.1	-1.1	---

For now, it is sufficient to know that a value noticeably larger than others in the matrix indicates an item pair that should be examined for possible LD.

The last new table is the result of the statement `HabermanResTbl=Yes;`. A portion of this table is reproduced below. In this table, observed response patterns are listed by row. For a given response pattern, the observed and model-implied frequencies, the standardized residual, EAP score, and standard deviation associated with the EAP score are printed.

**Output 2.16:** 1PL Grouped Data Output - Haberman Residual Table (Excerpt)

Response Pattern Observed and Expected Frequencies, Standardized Residuals, EAPs and SDs										
Group 1: Group1										
Item:					Frequencies		Standard			
1	2	3	4	5	Observed	Expected	Residual	EAP[q  u]	SD[q  u]	
0	0	0	0	0	3.00	2.37	0.41	-1.91	0.80	
0	0	0	0	1	6.00	5.47	0.23	-1.43	0.80	
0	0	0	1	0	2.00	2.47	-0.30	-1.43	0.80	
0	0	0	1	1	11.00	8.25	0.96	-0.94	0.81	
0	0	1	0	0	1.00	0.85	0.16	-1.43	0.80	
0	0	1	0	1	1.00	2.84	-1.09	-0.94	0.81	
...										
1	1	0	0	1	56.00	55.17	0.12	-0.44	0.82	
1	1	0	1	0	21.00	24.96	-0.80	-0.44	0.82	
1	1	0	1	1	173.00	177.91	-0.41	0.08	0.84	

1	1	1	0	0	11.00	8.59	0.83	-0.44	0.82
1	1	1	0	1	61.00	61.23	-0.03	0.08	0.84
1	1	1	1	0	28.00	27.71	0.06	0.08	0.84
1	1	1	1	1	298.00	295.80	0.15	0.63	0.86

## 2.8. The Nominal Categories Model

Our last set of basic examples will give demonstrations of using flexMIRT<sup>®</sup> to fit the reparameterized nominal categories model (Thissen, Cai, & Bock, 2010). As discussed in both Thissen and Steinberg (1988) and Thissen et al. (2010), models equivalent to the Partial Credit Model (Masters, 1982), and Muraki's (1992) Generalized Partial Credit Model, among others, may be obtained as restricted versions of the full-rank nominal model by the choice of contrast matrices and constraints.

### 2.8.1 Nominal Model

The first data set contains responses to items in a quality of life scale. First reported in Lehman (1988), the data set consists of responses from 586 patients to 35 items from the classic Quality of Life (QOL) Scale for the Chronically Mentally Ill. All items have 7 response categories: terrible; unhappy; mostly dissatisfied; mixed, about equally satisfied and dissatisfied; mostly satisfied; pleased; and delighted.

#### Example 2.7: Nominal Model Calibration

```

<Project>
Title = "QOL Data";
Description = "35 Items - Nominal";

<Options>
Mode = Calibration;
Etol = 1e-4;
Mtol = 1e-5;
Processors = 2;

<Groups>

%Group1%
File = "QOL.DAT";
Varnames = v1-v35;
N = 586;
Ncats(v1-v35) = 7;

```

```
Model(v1-v35) = Nominal(7);  
  
<Constraints>  
Fix (v1-v35),ScoringFn(1);
```

In the **<Groups>** section, we have identified the data file, named the variables, specified the number of response options, and chosen to fit the **Nominal** model with 7 categories to the data. In the **<Options>** section, the convergence criteria for both the E step and M iterations of the EM algorithm have been adjusted from the default values, by the **Etol** and **Mtol** commands, respectively, so that a tightly converged solution can be obtained.

Because the nominal categories model estimates parameters that differ from the models previously discussed, excerpts from the item parameter tables will be presented from the output and briefly discussed.

## Output 2.17: Nominal Model Output - Item Parameters

Nominal Model Slopes and Scoring Function Contrasts for Items for Group 1: Group1																								
Item	Label	P#	a	s.e.	Contrasts	P#	alpha 1	s.e.	P#	alpha 2	s.e.	P#	alpha 3	s.e.	P#	alpha 4	s.e.	P#	alpha 5	s.e.	P#	alpha 6	s.e.	
1	v1	6	0.83	0.17	Trend	1.00	----	1	-1.21	0.89	2	0.14	0.48	3	0.29	0.37	4	0.10	0.39	5	-0.19	0.37	6	0.37
2	v2	18	0.44	0.17	Trend	1.00	----	13	-1.01	1.71	14	0.06	0.95	15	-0.11	0.70	16	0.37	17	-0.12	0.61	18	0.61	
...																								
34	v34	402	0.49	0.13	Trend	1.00	----	397	-0.98	1.40	398	0.44	0.79	399	0.07	0.82	400	0.24	0.75	401	-0.13	0.56	402	0.56
35	v35	414	0.34	0.14	Trend	1.00	----	409	-0.97	2.17	410	0.22	1.17	411	0.14	1.11	412	0.20	0.93	413	-0.20	0.82	414	0.82
Nominal Model Scoring Function Values Group 1: Group1																								
Item	Label	s 1	s 2	s 3	s 4	s 5	s 6	s 7																
1	v1	0.00	0.79	1.15	1.31	3.09	4.38	6.00																
...																								
34	v34	0.00	1.10	1.44	1.82	3.10	3.92	6.00																
35	v35	0.00	0.92	1.35	1.69	3.32	4.18	6.00																
Nominal Model Intercept Contrasts for Items for Group 1: Group1																								
Item	Label	Contrasts	P#	gamma 1	s.e.	P#	gamma 2	s.e.	P#	gamma 3	s.e.	P#	gamma 4	s.e.	P#	gamma 5	s.e.	P#	gamma 6	s.e.				
1	v1	Trend	7	0.15	0.15	8	1.64	0.50	9	-0.54	0.28	10	0.27	0.21	11	0.14	0.20	12	0.09	0.17				
2	v2	Trend	19	0.34	0.14	20	0.64	0.52	21	-0.42	0.30	22	0.05	0.22	23	0.23	0.22	24	0.26	0.20				
...																								
34	v34	Trend	403	0.26	0.12	404	1.53	0.46	405	-0.70	0.30	406	0.16	0.29	407	-0.02	0.25	408	0.04	0.19				
35	v35	Trend	415	0.43	0.15	416	1.12	0.59	417	-0.77	0.37	418	0.15	0.27	419	0.15	0.29	420	0.11	0.27				
Original (Bock, 1972) Parameters, Nominal Items for Group 1: Group1																								
Item	Label	Category:	1	2	3	4	5	6	7															
1	v1	a	0.00	0.66	0.95	1.09	2.55	3.62	4.96															
		c	0.00	0.94	1.06	1.92	2.53	2.24	0.91															
2	v2	a	0.00	0.31	0.43	0.87	1.54	1.74	2.65															
		c	0.00	0.67	0.44	1.86	2.24	2.36	2.02															
...																								
34	v34	a	0.00	0.54	0.70	0.89	1.52	1.92	2.93															
		c	0.00	0.59	1.22	2.19	2.92	2.87	1.57															
35	v35	a	0.00	0.31	0.46	0.57	1.13	1.42	2.04															
		c	0.00	0.66	0.93	2.37	3.39	3.45	2.58															

The first 3 tables in Output 2.17 provide parameter estimates using the reparameterized version of the model (see Thissen et al., 2010). In the first table, the overall slope value, labeled **a**, the type of  $\alpha$  contrast matrix (**Ta**) used, and the estimated contrast values, labeled **alpha 1** – **alpha 3**, are reported. There are 7 categories so there are 6 scoring function contrasts. The second table, labeled **Nominal Model Scoring Function Values** reports the scoring function values for the categories, which are calculated as  $T_a\alpha$ , where  $\alpha$  are the estimated scoring contrasts. The third table of item parameters reports the type of contrast matrix used for the intercepts (**Tc**) and the estimated  $\gamma$  parameters. For convenience, the parameters are also reported in the original parameterization in the final table of the output presented.

The next example employs a set of data regarding numerical knowledge from pre-school aged children, most notably analyzed in Thissen and Steinberg (1988) to illustrate the concept of testlets. As described by Thissen and Steinberg (1988), the data were collected from 4 tasks that involved asking 592 young children to identify the numerals representing the quantities of 3 and 4 and, separately, match the numeral for each number to a correct quantity of blocks. From these items, called Identify3, Identify4, Match3, and Match4, pseudo-items were created that summarized a child's performance on both numbers. For the new pseudo-items, Identify and Match, the response options are 0 (child could do the requested action for neither number), 3 (child could identify/match 3 only), 4 (child could identify/match 4 only), and 34 (child could identify/match both 3 and 4 successfully). We will again fit the Nominal Model, but with some additional constraints imposed.

### Example 2.8: Nominal Model Calibration 2

```
<Project>
Title = "Pre-School Numerical Knowledge";
Description = "2 Redefined Items Nominal";
<Options>
  Mode = Calibration;
  SE = SEM;
  Etol = 5e-5;
  Mtol = 1e-9;
  GOF = Extended;
  M2 = Full;
  SX2Tbl = Yes;
  HabermanResTbl = Yes;
  FitNullModel = Yes;
```

```

    SavePRM = Yes;
<Groups>
%Group1%
    File = "PreschoolNum.dat";
    Varnames = Identify,Match,Identify3,Identify4,Match3,Match4;
    Select = Identify,Match;
    N = 592;
    Ncats(Identify,Match) = 4;
    Code(Identify,Match) = (0,3,4,34),(0,1,2,3);
    Model(Identify,Match) = Nominal(4);
    Ta(Identify,Match) = Identity;
    Tc(Identify,Match) = Identity;
<Constraints>
    Fix (Identify),ScoringFn(2);
    Equal (Match), ScoringFn(2) : Group1, (Match), ScoringFn(3);
    Equal Group1, (Match), Intercept(1) : Group1, (Match), Intercept(2);

```

Within the <Options> section, there are several new commands. The method of standard error calculation has been changed from the default of empirical cross-product approximation (**Xpd**) to Supplemented EM algorithm (keyword is **SEM**; see Cai, 2008). We have also requested the full version of the limited information GOF  $M_2$  statistic (Cai & Hansen, 2012; Maydeu-Olivares & Joe, 2005) and, additionally, specified that the zero-factor Null Model should be estimated and the associated incremental GOF indices be printed.

In the <Groups> section, we have identified the data file, named the variables, selected only the pseudo-items to retain for analysis, specified the number of response options, recoded the response options into zero-based sequential (but nevertheless nominal) codes, and chosen to fit the **Nominal** model with 4 categories to the data. The statements **Ta** and **Tc** are specific to the nominal model and refer to the contrast matrices for the scoring function and intercept parameters, respectively. The options for the scoring function and intercept contrasts include a Trend matrix (which uses a Fourier basis, augmented with a linear column), an Identity matrix, or a user-supplied matrix. In this example, both of the contrast matrices have been set to type **Identity** to facilitate equality constraints. Within the <Constraints> section, we encounter a more complicated set of constraints than in the previous example. Suffice it to say that these constraints are applied to the scoring functions and intercepts so that the solution matches with Thissen and Steinberg's (1988) preferred testlet solution. Estimated item parameters are presented in Output 2.18.

**Output 2.18:** Nominal Model 2 Output - Item Parameters

Nominal Model Slopes and Scoring Function Contrasts for Items for Group 1: Group1											
Item	Label	P#	a	s.e.	Contrasts	P# alpha 1	s.e.	P# alpha 2	s.e.	P# alpha 3	s.e.
1	Identify	2	0.80	0.61	Identity	1.00	----	0.00	----	1	1.21
2	Match	6	0.82	0.62	Identity	1.00	----	7	0.46	0.31	0.47
Nominal Model Scoring Function Values Group 1: Group1											
Item	Label	s 1	s 2	s 3	s 4						
1	Identify	0.00	0.00	1.21	3.00						
2	Match	0.00	0.46	0.46	3.00						
Nominal Model Intercept Contrasts for Items for Group 1: Group1											
Item	Label	Contrasts	P# gamma 1	s.e.	P# gamma 2	s.e.	P# gamma 3	s.e.			
1	Identify	Identity	3	-0.86	0.14	4	-0.80	0.43	5	0.65	0.20
2	Match	Identity	9	-0.42	0.28	9	-0.42	0.28	8	0.52	0.20
Original (Bock, 1972) Parameters, Nominal Items for Group 1: Group1											
Item	Label	Category:	1	2	3	4					
1	Identify	a	0.00	0.00	0.97	2.41					
		c	0.00	-0.86	-0.80	0.65					
2	Match	a	0.00	0.38	0.38	2.45					
		c	0.00	-0.42	-0.42	0.52					

Although several new command statements were introduced in this example, only two result in additions to the output file. Both the `M2` and `FitNullModel` statements provide additional GOF information that is printed near the bottom of the output file, with the other overall GOF index values. The fitted model's  $-2 \times \log$ -likelihood, AIC, and BIC are printed as before, but additional fit statistics for the zero-factor null model are also included under the corresponding fitted model section. The estimation of the independence model permits the calculation of the non-normed fit index (NNFI; aka Tucker-Lewis Index). The usual full-information fit statistics are printed, but limited-information statistics, based on  $M_2$ , for the fitted and zero-factor model are also given. Taken together, the fit indice values indicate that the nominal model provides excellent fit to this data.

**Output 2.19:** Nominal Model 2 Output - GOF values

Statistics based on the loglikelihood of the fitted model				
	-2loglikelihood:		2766.69	
	Akaike Information Criterion (AIC):		2784.69	
	Bayesian Information Criterion (BIC):		2824.14	
Statistics based on the loglikelihood of the zero-factor null model				
	-2loglikelihood:		2885.67	
	Akaike Information Criterion (AIC):		2895.67	
	Bayesian Information Criterion (BIC):		2917.58	
Full-information fit statistics of the fitted model				
	Degrees			
G2	of freedom	Probability	F0hat	RMSEA
8.45	6	0.2064	0.0143	0.03
	Degrees			
X2	of freedom	Probability	F0hat	RMSEA
8.99	6	0.1739	0.0152	0.03
Full-information fit statistics of the zero-factor null model				
	Degrees			
G2	of freedom	Probability	F0hat	RMSEA
127.43	10	0.0001	0.2153	0.14
	Degrees			
X2	of freedom	Probability	F0hat	RMSEA
124.58	10	0.0001	0.2104	0.14
With the zero-factor null model, Tucker-Lewis (non-normed) fit index based on G2 is 0.97				

With the zero-factor null model, Tucker-Lewis (non-normed) fit index based on X2 is 0.96				
Limited-information fit statistics of the fitted model				
	Degrees			
M2	of freedom	Probability	F0hat	RMSEA
0.17	6	0.9999	0.0003	0.00
Note: M2 is based on full marginal tables.				
Note: Model-based weight matrix is used.				
Limited-information fit statistics of the zero-factor null model				
	Degrees			
M2	of freedom	Probability	F0hat	RMSEA
117.87	10	0.0001	0.1991	0.13
Note: M2 is based on full marginal tables.				
Note: Model-based weight matrix is used.				
With the zero-factor null model, Tucker-Lewis (non-normed) fit index based on M2 is 1.09				

## 2.8.2 Generalized Partial Credit Model

We return the QOL data again to illustrate Muraki's (1992) Generalized Partial Credit (GPC) model. The GPC model is a constrained special case of the nominal model. To set up a GPC model using the reparameterized nominal model, one fixes all scoring function contrasts. This results in the first scoring function contrast being fixed to 1.0 and the other contrasts fixed to 0.0. With such constraints in place, the category scoring function values are equal to  $0, 1, \dots, k$ , which is the necessary structure for fitting the GPC. Because the GPC is such a popular special case of the nominal model in flexMIRT<sup>®</sup> 2.0 we have added `GPC()` as an option to the `Model()` statement. This option will automatically fix the scoring functions as needed, freeing the user from making such technical specifications. The following syntax demonstrates the use of this new model keyword. As with the `Graded` model specification, when one uses the `GPC()` keyword in the `Model` statement, the model name is immediately followed by the number of categories in parentheses.

### Example 2.9: Generalized Partial Credit Model

<Project>
Title = "QOL Data";

```

Description = "35 Items - GPC";
<Options>
  Mode = Calibration;
  Etol = 1e-4;
  Mtol = 1e-5;
  Processors = 2;
<Groups>
%Group1%
  File = "QOL.DAT";
  Varnames = v1-v35;
  N = 586;
  Ncats(v1-v35) = 7;
  Model(v1-v35) = GPC(7);
<Constraints>

```

### 2.8.3 Rating Scale Model

Andrich's (1978) Rating Scale (RS) model may also be obtained as a special case of the reparameterized nominal model. To obtain the RS model in flexMIRT<sup>®</sup> it is necessary to fix the scoring functions for all items. Additionally, we need to constrain the slopes for all items to be equal, as well as set each intercept to be equal across the items. This is all accomplished via the constraints imposed in Example 2.10.

#### Example 2.10: Rating Scale Model with Trend Contrasts

```

<Project>
Title = "QOL Data";
Description = "35 Items RS Using Default Trend Contrasts";

<Options>
  Mode = Calibration;
  SE = SEM;

<Groups>

%Group1%
  File = "QOL.DAT";
  Varnames = v1-v35;

```

```

N = 586;
Ncats(v1-v35) = 7;
Model(v1-v35) = Nominal(7);

<Constraints>
  Fix (v1-v35),ScoringFn;
  Equal (v1-v35),Slope;
  Equal (v1-v35),Intercept(2);
  Equal (v1-v35),Intercept(3);
  Equal (v1-v35),Intercept(4);
  Equal (v1-v35),Intercept(5);
  Equal (v1-v35),Intercept(6);

```

The previous RS fitting used the flexMIRT<sup>®</sup> default trend contrasts. To obtain results for the RS model similar to those that would be produced by Multilog, the user may provide flexMIRT<sup>®</sup> with the triangle contrasts utilized by Multilog. This is done through the Tc command and, rather than using one of the available keywords, supplying the values that compose the desired contrast. The triangle contrast is shown in the Tc statement in the following example syntax.

### Example 2.11: Rating Scale Model with Triangle Contrasts

```

<Project>
Title = "QOL Data";
Description = "35 Items RS Multilog-style Triangle Contrasts";

<Options>
  Mode = Calibration;
  SE = SEM;

<Groups>

%Group1%
  File = "QOL.DAT";
  Varnames = v1-v35;
  N = 586;
  Ncats(v1-v35) = 7;
  Model(v1-v35) = Nominal(7);
  Tc(v1-v35) = (

```

```

    0  0  0  0  0  0  0,
    -1 -1  0  0  0  0  0,
    -2 -1 -1  0  0  0  0,
    -3 -1 -1 -1  0  0  0,
    -4 -1 -1 -1 -1  0  0,
    -5 -1 -1 -1 -1 -1  0,
    -6  0  0  0  0  0  0 );

<Constraints>
    Fix (v1-v35),ScoringFn;
    Equal (v1-v35),Slope;
    Equal (v1-v35),Intercept(2);
    Equal (v1-v35),Intercept(3);
    Equal (v1-v35),Intercept(4);
    Equal (v1-v35),Intercept(5);
    Equal (v1-v35),Intercept(6);

```

Thus far, we have covered how to use flexMIRT® to calibrate several of the most commonly used single-group unidimensional IRT models, and to obtain IRT scale scores. While these types of models are useful, the greatest strength of flexMIRT® is its ability to readily handle complex models (e.g., multidimensional, bifactor, testlet, etc.) and more complex data structures, such as multilevel data. In the next chapter, we will provide instruction and examples, building intuitively from the basic syntax already presented, to demonstrate how flexMIRT® syntax files are constructed to handle more complex models and data structures.

---

### Advanced Modeling

---

The previous chapter focused on basic unidimensional, single group models to introduce the flexMIRT<sup>®</sup> syntax, output, and available IRT models. While these examples are useful in their own right, it is expected that a user working with real-world data will need more advanced modeling options. In this chapter, we will provide examples for using flexMIRT<sup>®</sup> when multiple groups, multiple factors, and/or a hierarchical data structure need to be modeled.

---

#### 3.1. Multiple Group Calibration

The data set used to illustrate 2PL and 3PL examples in the previous chapter is actually made up of responses from two groups of examinees, 3rd and 4th graders. Using this already familiar data set, we will demonstrate multiple group analysis.

**Example 3.1:** 2-Group 3PL Using Normal Priors with EAP Scoring

```
<Project>
Title = "G341-19";
Description = "12 Items 3PL 2 Groups
Calibration with Normal Prior, Saving Parameter Estimates,
Followed by Combined Scoring Run for EAP Scores";
<Options>
Mode = Calibration;
Progress = Yes;
SaveSCO = Yes;
SavePRM = Yes;
SaveInf = Yes;
Score = EAP;
FisherInf = 81,4.0;
SE = Xpd;
```

```

GOF = Extended;
M2 = Full;
FitNullModel = Yes;
<Groups>
%Grade4%
File = "g341-19.grp1.dat";
Varnames = v1-v12;
N = 1314;
Ncats(v1-v12) = 2;
Model(v1-v12) = ThreePL;
BetaPriors(v1-v12) = 1.5;
%Grade3%
File = "g341-19.grp2.dat";
Varnames = v1-v12;
N = 1530;
Ncats(v1-v12) = 2;
Model(v1-v12) = ThreePL;
BetaPriors(v1-v12) = 1.5;
<Constraints>
Free Grade3, Mean(1);
Free Grade3, Cov(1,1);
Equal Grade3, (v1-v12), Guessing : Grade4, (v1-v12), Guessing;
Equal Grade3, (v1-v12), Intercept: Grade4, (v1-v12), Intercept;
Equal Grade3, (v1-v12), Slope : Grade4, (v1-v12), Slope;
Prior Grade3, (v1-v12), Guessing : Beta(1.0,4.0);

```

Two groups (%Grade4% and %Grade3%) are specified in the <Groups> section. An important point to highlight is that the data for each group should be stored as individual files. To that end, “g341-19.dat” is split into 2 new files “g341-19.grp1.dat” and “g341-19.grp2.dat”. As may be seen in the N = statements for each group, the original sample size of 2844 is now divided between the two groups. Within each group the basic specification statements (VarNames, Ncats, etc) remain the same. There are now two sets of them, one per group. A new option not specific to multiple group analysis, is **BetaPriors**. This statement imposes a prior on the item uniquenesses in the form of a  $\text{Beta}(\alpha, 1.0)$  distribution with a user-specified  $\alpha$  parameter, in this case  $\alpha = 1.5$  (see Bock, Gibbons, & Muraki, 1988).

Within the **<Constraints>** section, we are allowing the mean and variance of the second group (Grade3) to be estimated by freeing those parameters via the first two constraint statements. Additionally, we have specified a model that assumes measurement invariance by constraining the item parameters to be equal across the two groups. Finally, we supplied a Beta distribution prior for the guessing parameters.

There are new commands encountered in the **<Options>** section as well. It should be noted that these newly encountered commands are not specific in any way to the multi-group format and are available for use with single group analyses as well. For instance, the **Progress** statement (via **Yes/No** keywords) determines whether flexMIRT® will print progress updates in the console window. Using **SaveInf= Yes;**, we have requested that the Fisher information function for the items and the total scale be saved to an external file and have specified via **FisherInf = 81,4.0;** that we would like information values calculated at 81 points, equally spaced from -4.0 to 4.0. The method of standard error calculation has been specified as empirical cross-products approximation, using **SE = Xpd** (all possible SE calculation methods and corresponding keywords are covered in Chapter 5: Details of the Syntax).

---

### 3.2. Characterizing Non-Normal Population Distributions

This example is substantially the same as the analysis just discussed, with an additional option included in the calibration portion. As the command file does not change drastically, only the modified sections are presented here. The full syntax file (g341-19.calib.EH.flexmirt) is available in the examples provided on the flexMIRT® website.

#### Example 3.2: 2-Group 3PL with Empirical Histogram (Excerpts)

```
...
<Options>
...
Etol = 5e-4;
<Groups>
%Grade4%
File = "g341-19.grp1.dat";
Varnames = v1-v12;
N = 1314;
Ncats(v1-v12) = 2;
```

```

Model(v1-v12) = ThreePL;
BetaPriors(v1-v12) = 1.5;
EmpHist = Yes;
%Grade3%
File = "g341-19.grp2.dat";
Varnames = v1-v12;
N = 1530;
Ncats(v1-v12) = 2;
Model(v1-v12) = ThreePL;
BetaPriors(v1-v12) = 1.5;
EmpHist = Yes;
...

```

The primary point of interest here is the `EmpHist = Yes;` statement found as the last command provided for each group. With this option invoked, flexMIRT<sup>®</sup> will estimate the item parameters and the population (theta) distribution simultaneously. The latent variable distribution is estimated as an empirical histogram (e.g., Mislevy, 1984; Woods, 2007). By “empirical histogram” we mean the normalized accumulated posterior densities for all response patterns at each quadrature node - that is, an empirical characterization of the shape of the examinee ability distribution in the population. The use of an empirical histogram is in contrast to a typical calibration run in which the latent trait is assumed to be normally distributed. This estimation method is currently only available for single-level unidimensional and single-level bifactor (or testlet response models) with one general dimension. In the `<Options>` section, the convergence criterion for the E step of the EM algorithm has been increased from the default of 1e-4 to 5e-4 to accommodate the added uncertainty due to the estimation of the empirical histogram prior.

### 3.3. Multiple Group Scoring Only

Scoring runs with multiple groups proceed in much the same way as the single group scoring analyses. The current scoring example makes use of the item parameters obtained from the first multiple group example (Example 3.1 which employed normal priors). The syntax is presented below and, as with the single group scoring syntax, in the `<Options>` section `Mode` is set to `Scoring`, the type of scores requested is specified (MAPs in this case) and the file containing the relevant item parameters (generated by our previous calibration run) is given using the `ReadPRMFile` statement. Because of the distinct groups, two sets of data descriptor statements (e.g., syntax for declaring group label, data file

name, variable names, and sample size) are given. Because this example is so similar to the single group scoring runs, no further discussion is provided. For explications of any of the included syntax statements, the user should refer to a previous single-group example (e.g., Example 2.3).

**Example 3.3:** 2-Group 3PL MAP Scoring from Parameter Estimates

```
<Project>
Title = "G341-19";
Description = "12 Items 3PL 2 Groups MAP from Parameter Estimates";
<Options>
Mode = Scoring;
ReadPRMFile = "g341-19.calib.normal-prm.txt";
Score = MAP;
<Groups>
%Grade4%
File = "g341-19.grp1.dat";
VarNames = v1-v12;
N = 1314
%Grade3%
File = "g341-19.grp1.dat";
VarNames = v1-v12;
N = 1530
<Constraints>
```

It some situations, it may be useful to obtain the sum score conversion table, without actually having to score data. For example, when only the SSC table is wanted and the sample size is large enough that calculating scores for every individual would take a non-trivial amount of computing time, flexMIRT<sup>®</sup> is able to by-pass individual scoring and produce only the necessary table. The syntax to do so is straight-forward and presented below.

**Example 3.4:** 2-Group 3PL Summed Score to EAP Conversion Table

```
<Project>
Title = "G341-19";
Description = "12 Items 3PL 2 Groups Summed Score to EAP from
Parameter Estimates Using Estimated Empirical Histogram Prior
Make Table Only";
<Options>
```

```

Mode = Scoring;
ReadPRMFile = "g341-19.calib.EH-prm.txt";
Score = SSC;
<Groups>
% Grade4 %
Varnames = v1-v12;
% Grade3 %
Varnames = v1-v12;
<Constraints>

```

Just as a typical scoring run, we have specified the parameter file (now using parameters obtained from the multiple-group calibration which estimated theta via an empirical histogram). We have set the scoring method to produce the desired SSC (Summed Score to EAP Conversion) table. In the **<Groups>** section, we still provide labels and variable names for both groups, but the sample size and data file names are omitted because they are not needed for producing the conversion table without scoring any individuals.

### 3.4. DIF Analyses

Up to this point, the multiple group examples have assumed the invariance of item parameters across groups. However, when multiple groups are involved, it is generally prudent to check that assumption by looking for differential item functioning (DIF) between groups. flexMIRT<sup>®</sup> provides two general methods for DIF testing (Langer, 2008): 1) a DIF sweep for all items, and 2) a more focused examination, where candidate DIF items are tested with designated anchor items. Both methods utilize the Wald test. We will first cover the more general DIF sweep, again using the end-of-grade testing data. As the syntax is quite similar to what was presented earlier, only the relevant changes will be noted in the excerpt. The full syntax file may be found in the corresponding folder on the flexMIRT<sup>®</sup> support website.

#### Example 3.5: DIF: Test All Items (Excerpts)

```

...
<Options>
...
DIF = TestAll;
DIFcontrasts = (1.0 -1.0);
...
<Constraints>

```

```

...
Prior Grade3, (v1-v12), Guessing : Normal(-1.1,0.5);
Prior Grade4, (v1-v12), Guessing : Normal(-1.1,0.5);

```

First, in conjunction with the use of the 3PL model, a logit-normal prior is applied to the guessing parameters, which was a previously unseen option. With respect to the DIF analysis, the relevant new statements are contained in the <Options> section. To enable the general DIF sweep, testing all items, `DIF = TestAll;` is specified. Also required when testing for DIF are values that supply flexMIRT<sup>®</sup> with the entries for a DIF contrast matrix, specifying how contrasts among groups should be constructed. In this example, we are conducting a straight-forward comparison of Grade3 against Grade4, so the statement appears as, `DIFcontrasts = (1.0 -1.0);` with spaces between the supplied values for each contrast. When  $k$  groups are present,  $k - 1$  contrasts must be specified, (e.g., for 3 groups, `DIFcontrasts = (2.0 -1.0 -1.0, 0.0 1.0 -1.0);`, with commas in between). Because the syntax file is free-form, the contrasts may appear on more than one line. For example,

```

DIFcontrasts = (
2.0 -1.0 -1.0,
0.0 1.0 -1.0
);

```

is entirely permissible.

From the DIF analysis, an additional table is presented in the output.

**Output 3.1: DIF Analyses Output Table**

DIF Statistics for 3PL Items											
Item numbers in:											
Grp1	Grp2	Total	X2	d.f.	p	X2g	d.f.	p	X2a g	d.f.	p
1	1	1.8	3	0.6081	0.1	1	0.8141	1	0.2371	0.4	1
2	2	8.5	3	0.0370	0.2	1	0.6717	1	0.0280	3.5	1
3	3	0.5	3	0.9177	0.0	1	0.9096	1	0.5425	0.1	1
4	4	17.1	3	0.0007	0.0	1	0.8647	1	0.5910	16.8	1
5	5	1.8	3	0.6053	0.1	1	0.7717	1	0.2731	0.6	1
6	6	0.6	3	0.8922	0.1	1	0.7909	1	0.5058	0.1	1
7	7	0.7	3	0.8622	0.0	1	0.8730	1	0.6061	0.5	1
8	8	8.8	3	0.0317	0.0	1	0.9159	1	0.0067	1.5	1
9	9	3.6	3	0.3093	0.2	1	0.6730	1	0.5115	3.0	1
10	10	1.5	3	0.6779	0.0	1	0.9055	1	0.4001	0.8	1
11	11	1.9	3	0.5977	0.9	1	0.3405	1	0.9424	1.0	1
12	12	0.8	3	0.8486	0.5	1	0.4906	1	0.9307	0.3	1

For each item that is present in both groups, a line in the table is added that reports the various Wald tests for that item. As we fit the 3PL to the items, we are provided with an overall test for DIF, a test of DIF with respect to the guessing parameter (labeled **X2g**), a test of the slope, assuming  $g$  is fixed across groups (labeled **X2a|g**), and a test of the intercept, assuming  $a$  and  $g$  are fixed. Each of the reported Wald values also has the associated degrees of freedom and p-value printed. Woods, Cai, and Wang (2013) found that the Wald-2 procedure, implemented via **DIF = TestAll;** in flexMIRT<sup>®</sup>, tends to result in inflated Type I error rates and suggested that this procedure may be best used for identifying anchors for a subsequent DIF analysis that examines only candidate items for DIF, rather than as a final DIF analysis from which conclusions regarding item functioning are drawn. Additionally, it is also recommended that when using the **TestAll** DIF sweep option that the standard error method be changed from the default. Conclusions reported in Woods et al. (2013) were based on the use of supplemented EM SEs (**SE = SEM;**).

flexMIRT<sup>®</sup> may also be used for more traditional DIF testing of test candidate items against designated anchor items. Only relevant excerpts of the command file, "g341-19.difcand.flexmirt", are presented. Again, the full file is available on the support website.

### Example 3.6: DIF: Test Candidate Items (Excerpts)

```
...
<Options>
...
DIF = TestCandidates;
DIFcontrasts = (1.0 -1.0);
<Groups>
%Grade4%
File = "g341-19.grp1.dat";
VarNames = v1-v12;
N = 1314;
Ncats(v1-v12) = 2;
Model(v1-v12) = ThreePL;
DIFitems = v1,v2;
%Grade3%
File = "g341-19.grp2.dat";
VarNames = v1-v12;
```

```

N = 1530;
Ncats(v1-v12) = 2;
Model(v1-v12) = ThreePL;
DIFitems = v1,v2;

<Constraints>
Free Grade3, Mean(1);
Free Grade3, Cov(1,1);
Equal Grade3, (v3-v12), Guessing : Grade4, (v3-v12), Guessing;
Equal Grade3, (v3-v12), Intercept: Grade4, (v3-v12), Intercept;
Equal Grade3, (v3-v12), Slope      : Grade4, (v3-v12), Slope;
...

```

The keyword used with the DIF statement is now `TestCandidates`, as opposed to `TestAll` in the previous example. The `DIFcontrasts` statement remains unchanged. To specify candidate DIF items, within the `<Groups>` section, we simply list the candidate item names using the `DIFitems` statement, found as the last entry within each group subsection of the excerpted code. Here, we are testing Items 1 and 2 for possible DIF. As can be seen in the `<Constraints>` section, the item parameters for the anchor items (`v3-v12`) are set equal across groups. The DIF table printed in the output is structured as in the previous example and will not be covered here.

---

### 3.5. Multiple Group Multilevel Model

Our final multiple group example will cover the flexMIRT<sup>®</sup> syntax options for handling nested data. Using the student cognitive outcomes data from the 2000 Program for International Student Assessment (see Adams & Wu, 2002), 31 mathematics items with mixed formats (multiple choice and constructed response) from students in both the United States and Ireland will be analyzed. The US data set is comprised of 2115 students, representing 152 different schools, and the Irish data set has 2125 student observations, coming from 139 different schools. From the general description of the data, it is clear that the complete set of data for this examples has two groups (US and Irish students) and within each country, students (level-1 units) are nested within schools (level-2 units). Because the full command file (PISA00mathL2.flexmirt) is quite long, only commands relevant to multilevel aspect of the analysis will be highlighted.

### Example 3.7: Multiple Group Multilevel Model (Excerpts)

```
...
<Groups>

%USA%
  File = "PISA00Math.US.dat";
  Missing = 9;
  Varnames = SchID,StdID,
    ViewRoomQ1,
    BricksQ1,
  ...
    CarpenterQ01,
    PipelinesQ1;
  Select = ViewRoomQ1,...,PipelinesQ1;
  CaseID = StdID;
  Cluster = SchID;
  Dimensions = 2;
  Between = 1;
  N = 2115;
  ...
%Ireland%
  File = "PISA00Math.IE.dat";
  ...
  CaseID = StdID;
  Cluster = SchID;
  Dimensions = 2;
  Between = 1;
  N = 2125;
  ...
<Constraints>
  ...
  Free USA,Cov(1,1);
  Free Ireland,Mean(1);
  Free Ireland,Cov(1,1);
```

The data files are specified as usual within each group. We have also set a missing data code using `Missing = 9;`, so all 9s in the data set will be interpreted as missing values, rather than the default missing value of -9. The variables are named in the typical fashion; the first two variables are of note because

they indicate student ID numbers and school ID numbers, respectively, and will not be selected for IRT analysis. In general, a multilevel analysis has two kinds of latent dimensions, Between and Within. The Between dimensions will always precede the Within dimensions when flexMIRT<sup>®</sup> is assigning dimensions. For this example, there are only two dimensions total so we specify the total number of dimensions to be used for the model (via `Dimensions=2;`) and also denote that at level-2 there is 1 between-school dimension (`Between = 1;`). This implies that there is 1 within-school dimension at the student level. In the estimation and output, flexMIRT<sup>®</sup> will use the first dimension for between-school variations and the second dimension for within-group variations.

When a 2-level model has been requested, it is required that a variable defining the higher-level units be specified via the `Cluster` statement. We also provide a variable that identifies the first level data (individuals) with the `CaseID` statement. The latter can be convenient if the individual IDs are unique so that the scale scores computed by flexMIRT<sup>®</sup> can be merged into other data sets for further analysis.

The `<Constraints>` section contains extensive statements for setting up the necessary priors and equality constraints both within and across the two groups so that the desired hierarchical model is estimated. With the between-group equality constraints, the item parameters (for both level-1 and level-2 models) are set equal across the two countries, thereby creating anchoring so that the mean of Ireland schools can be freely estimated. The within-group equality constraints are set up so that the item slopes for the within dimension is equal to those for the between dimension. This implies that a random-intercept only model is fitted in each country, which permits the decomposition of between-school versus within-school variance.

### Output 3.2: Two-Level Model - Means and Variances

```

...
PISA 2000 Math Data: Two-Level Unidimensional
31 Items
US Data: 2115 Students, 152 Schools
IE Data: 2125 Students, 139 Schools

Summary of the Data and Control Parameters
      Group      USA      Ireland
Missing data code      9      9
Number of Items      31      31
Number of L-2 Units      152      139
Number of L-1 Units      2115      2125
Number of Dimensions      2      2
Between Dimensions      1      1
Within Dimensions      1      1
...
Group Parameter Estimates:
      Group      Label      P#      mu 1      s.e.      P#      mu 2      s.e.
      1      USA      79      0.00      ----      0.00      ----
      2      Ireland      79      0.36      0.34      0.00      ----

Latent Variable Variance-Covariance Matrix for Group1: USA
P# Theta 1      s.e.      P# Theta 2      s.e.
1      0.52      0.07
      0.00      ----      1.00      ----

Latent Variable Variance-Covariance Matrix for Group2: Ireland
P# Theta 1      s.e.      P# Theta 2      s.e.
80      0.11      0.02
      0.00      ----      1.00      ----

```

We can see from the output above that the summary of data shows the multilevel syntax has been correctly interpreted. On average, the overall school-level achievement in Ireland is 0.36 standard deviations higher than that of the US. The more interesting result, however, is in the variance decompositions. In the US, with a fixed within-school variance of 1.0 (for identification), the between-school variance is estimated as 0.52. This implies an intra-class correlation of  $0.52/(1 + 0.52) = 0.34$ , showing a significant amount of school level variations in the US. On the other hand, the intra-class correlation for Ireland is equal to  $0.11/(1 + 0.11) = 0.10$ , which implies that the school achievement-levels are much more homogeneous.

---

### 3.6. Multidimensional Model - 4 Correlated Factors

Although the previous example was multidimensional, estimating two factors, it was also dealing with nested/hierarchical data. Because the fitting of a multidimensional model was mixed in with the hierarchical modeling syntax, we provide a more straight-forward multidimensional example by fitting a model with four correlated latent factors to a single group. The QOL dataset was introduced in Chapter 2, but a previously unmentioned aspect of the scale is it was designed to assess both the overall quality of life as well as the quality of life in 7 more specific subdomains (family, money, health, leisure, living, safety, and social), with each subdomain containing 4 to 6 items. Item 1 is a global quality of life item. We will be fitting a four factor model to a subset of the items of the previously introduced QOL scale (items from the family, money, health, and leisure factors) and allowing the latent factors to correlate.

#### Example 3.8: Four Factor GRM Model with Correlated Factors

```
<Project>
<Project>
Title = "QOL Data";
Description = "Items 2-21 with Correlated Factors";

<Options>
  Mode = Calibration;
  Quadrature = 21,5.0;
  SavePRM = Yes;
  Processor = 4;
  FactorLoadings = Yes;

<Groups>

%Group1%
File = "QOL.DAT";
Varnames = v1-v35;
Select = v2-v21;

N = 586;

Dimensions = 4;
```

```

Ncats(v2-v21) = 7;
Model(v2-v21) = Graded(7);

<Constraints>
  Fix (v2-v21),Slope;

  Free (v2-v5),Slope(1);
  Free (v6-v9),Slope(2);
  Free (v10-v15),Slope(3);
  Free (v16-v21),Slope(4);

  Free Cov(2,1);

  Free Cov(3,1);
  Free Cov(3,2);

  Free Cov(4,1);
  Free Cov(4,2);
  Free Cov(4,3);

```

Points of note in the syntax, relevant to the fitted multidimensional model are the **Dimensions** statement in the **<Groups>** section and the various constraints encountered in the **<Constraints>** section. As described earlier, we are fitting a model with four factors, hence the **Dimensions = 4;** statement. Within the **<Constraints>** section, we are assigning the items to the appropriate factors.

The first step in assigning items to factors to fix all slope values to 0, via **Fix (v2-v21),Slope;**, which allows us, in the next step, to free the individual items to load on only the appropriate factors. As can be seen in the four **Free** statements, we are specifying that Items 2 through 5 load on the first factor, Items 6 through 9 load on the second factor, Items 10 through 15 load are assigned to the third factor, and so on. This corresponds to the description provided earlier, in which Items 2 to 5 belong to the “Family” factor only and Items 6 to 9 belong to only the “Money” factor, etc. The final constraints, such as **Free Cov(2,1);**, are used to tell flexMIRT® to freely estimate the covariance/correlation between the factors. Using the lower triangle of the variance/covariance matrix of the latent factors to determine positions, the variance of the first factor is found at Cov(1,1), the variance of the second factor at Cov(2,2) and the covariance of the two factors is located in Row 2, Column 1 (i.e., Cov(2,1)) of the matrix and so on.

Additionally, in the <Options> section, the **Quadrature** command is encountered for the first time. This statement allows the user specify the number of quadrature points to be used, 21 in this case, and the range over which those points should be spread (from -5.0 to 5.0 here). Reducing the number of quadrature points from the default value (49 points per dimension) may be necessary when fitting multidimensional models because the total number of points used is exponentially related to the number of dimensions. With higher dimensional models, the total number of quadrature points (and therefore processing time) may become unwieldy if the default value is maintained.

Within the output, the various sections are arranged as with the other examples. In the output file, the factor loadings are printed below the estimated item parameters, as requested by the **FactorLoadings = Yes;** statement. As seen before, the group means of the latent variables are reported followed by the estimated latent variable variance/covariance matrix. This section of the output is presented below.

### Output 3.3: Correlated Four Factor Model Output - Loadings, Means and Variances

Factor Loadings for Group 1: Group1												
Item	Label	lambda 1	s.e.	lambda 2	s.e.	lambda 3	s.e.	lambda 4	s.e.			
1	v2	0.83	0.03	0.00	----	0.00	----	0.00	----			
2	v3	0.78	0.03	0.00	----	0.00	----	0.00	----			
3	v4	0.93	0.02	0.00	----	0.00	----	0.00	----			
4	v5	0.92	0.02	0.00	----	0.00	----	0.00	----			
5	v6	0.00	----	0.87	0.03	0.00	----	0.00	----			
6	v7	0.00	----	0.72	0.05	0.00	----	0.00	----			
7	v8	0.00	----	0.90	0.02	0.00	----	0.00	----			
8	v9	0.00	----	0.88	0.03	0.00	----	0.00	----			
9	v10	0.00	----	0.00	----	0.66	0.06	0.00	----			
10	v11	0.00	----	0.00	----	0.74	0.05	0.00	----			
11	v12	0.00	----	0.00	----	0.71	0.05	0.00	----			
12	v13	0.00	----	0.00	----	0.72	0.05	0.00	----			
13	v14	0.00	----	0.00	----	0.78	0.04	0.00	----			
14	v15	0.00	----	0.00	----	0.72	0.05	0.00	----			
15	v16	0.00	----	0.00	----	0.00	----	0.77	0.04			
16	v17	0.00	----	0.00	----	0.00	----	0.73	0.04			
17	v18	0.00	----	0.00	----	0.00	----	0.76	0.04			
18	v19	0.00	----	0.00	----	0.00	----	0.84	0.03			
19	v20	0.00	----	0.00	----	0.00	----	0.75	0.04			
20	v21	0.00	----	0.00	----	0.00	----	0.49	0.07			

QOL Data

Items 1-21 Correlated Factors with Correlated Factors

Group Parameter Estimates:

Group	Label	P#	mu 1	s.e.	P#	mu 2	s.e.	P#	mu 3	s.e.	P#	mu 4	s.e.
1	Group1		0.00	----		0.00	----		0.00	----		0.00	----

Latent Variable Variance-Covariance Matrix for Group1: Group1

P#	Theta 1	s.e.	P#	Theta 2	s.e.	P#	Theta 3	s.e.	P#	Theta 4	s.e.
	1.00	----									
141	0.41	0.04		1.00	----						
142	0.55	0.03	143	0.50	0.04		1.00	----			
144	0.55	0.04	145	0.56	0.03	146	0.69	0.03		1.00	----

The pattern of the estimated loadings indicates that the items were assigned to the factors as expected, with certain loadings restricted to 0.00 and having no associated SE. Because there is only a single group, the latent variable means were also restricted to zero and have no associated SE, indicating they were not estimated. The final section in the output excerpt reports the correlation matrix of the latent variable - for example, the correlation between the first two standardized factors is estimated to be 0.41, with a SE of 0.04.

---

### 3.7. The Bifactor Model

The item bifactor model is a special multidimensional IRT model, in which each item is restricted to load on, at most, two factors. The typical set up for a bifactor model is one factor on which all the items load, termed the general factor, and several more specific factors, termed group-specific factors, onto which only subsets of the items can load. A common substantive example of this would be a reading test made up of several short reading passages that each have several items probing the students comprehension for a particular passage. The general factor in this case would be “Reading” and each passage would have its own specific factor, onto which only the items related to that passage load.

Note that Bradlow, Wainer, and Wang’s (1999) testlet response theory model can be understood as a further restricted bifactor model with within-item proportionality constraints on the general and group-specific slopes (see also, Glas, Wainer, & Bradlow, 2000). With generalized capability to impose constraints, flexMIRT<sup>®</sup> can estimate testlet models too. Chief among the benefits of the item bifactor model is computational efficiency in which full-information maximum marginal likelihood estimation of item parameters can proceed with 2-dimensional integration regardless of how many group-specific factors are in the model, due to a result first discovered by Gibbons and Hedeker (1992). The bifactor model in flexMIRT<sup>®</sup> is in fact obtained as a two-tier model (Cai, 2010a). flexMIRT<sup>®</sup> is fully capable of multiple-group bifactor or two-tier modeling as detailed in Cai, Yang, and Hansen (2011).

#### 3.7.1 Bifactor Model Using the Reparameterized Nominal Categories Model

Here, we use the full 35 item Quality of Life data to illustrate a bifactor analysis, specifying a general QOL factor and 7 specific factors to capture the subdomains mentioned earlier. Gibbons et al. (2007) reported a graded bifactor analysis of this data set using the original 7 categories and we will

replicate this structure but employ a full-rank nominal model.

**Example 3.9:** Bifactor Structure - Nominal Model

```
<Project>
Title = "QOL Data";
Description = "35 Items Bifactor Nominal Model";

<Options>
  Mode = Calibration;
  Quadrature = 21,5.0;
  Etol = 1e-4;
  Mtol = 1e-5;
  Processors = 2;
<Groups>
%Group1%
  File = "QOL.DAT";
  Varnames = v1-v35;
  N = 586;
  Ncats(v1-v35) = 7;
  Model(v1-v35) = Nominal(7);
  Dimensions = 8;
  Primary = 1;
<Constraints>
  Fix (v1-v35),ScoringFn(1);

  Fix (v1-v35),Slope;
  Free (v1-v35),Slope(1);
  Free (v2-v5),Slope(2);
  Free (v6-v9),Slope(3);
  Free (v10-v15),Slope(4);
  Free (v16-v21),Slope(5);
  Free (v22-v26),Slope(6);
  Free (v27-v31),Slope(7);
  Free (v32-v35),Slope(8);
```

With regards to the bifactor structure, in the **<Groups>** section, a multidimensional model with 8 dimensions is first specified. This is the total number of domains/factors, both general and specific. The **Primary** command specifies, out of the total number of dimensions, how many are primary/general

dimensions. As noted in the data description, the QOL scale is made of 1 general domain and 7 subdomains.

Once the dimensionality of the model is set up in the `<Groups>` section, the details of which items load onto which specific factors are given in the `<Constraints>` section. The first constraint serves to fix all item slopes to zero, facilitating the subsequent freeing of parameters. The next 8 lines set up the factor structure by 1) freeing all the item slopes on the first dimension, which will be the general factor, and 2) assigning the items to the appropriate subdomains, based on the item content. *Note that it is absolutely necessary to ensure the the primary dimension(s) precede the group-specific dimensions when freeing items to load on the factors.* flexMIRT<sup>®</sup> automatically reduces the dimensionality of integration to 2 and consequently the analysis runs extremely fast.

The `Model(v1-v35) = Nominal(7);` statement in the `<Groups>` section indicates that we wish to fit the Nominal Model (for 7 response categories) to the items. In the current code, we have opted not to modify either the scoring functions contrasts (which may accomplished via the `Ta = ;` statement) or the intercept contrast matrix (which may accomplished via the `Tc = ;` statement). Without changes, the default option of Trend matrices will be used for both sets of parameters. In the `<Constraints>` section, the first constraint imposed fixes the first scoring function contrast for all 35 items (the default value is 1.0), as required for identification (see Thissen et al., 2010). With the default contrast matrices, the fixed first scoring function contrast of 1.0 ensures that the first and last scoring function values are fixed to 0 and 6, respectively. The remaining 5 scoring function values are reparameterized by the contrast matrices and become estimatable parameters (as shown below).

In the `<Options>` section, the number of quadrature points is reduced for time-efficiency, as was done with the other MIRT models, using the `Quadrature` statement and the tolerance values are adjusted down to obtain tighter solutions.

### Output 3.4: Nominal Model Scoring Function Estimates - Excerpt

Nominal Model Scoring Function Values (s) Group 1: Group1									
Item	Label	s 1	s 2	s 3	s 4	s 5	s 6	s 7	
1	v1	0.00	0.76	1.18	1.46	3.29	4.61	6.00	
2	v2	0.00	0.79	1.53	2.58	4.18	4.77	6.00	
3	v3	0.00	1.49	1.91	2.08	3.77	4.21	6.00	
4	v4	0.00	0.77	1.65	2.48	4.05	4.76	6.00	
5	v5	0.00	0.99	1.71	2.47	3.93	4.78	6.00	
6	v6	0.00	0.93	2.46	3.13	4.35	5.12	6.00	

7	v7	0.00	0.79	2.27	2.96	4.25	5.03	6.00
8	v8	0.00	1.50	2.30	2.92	4.27	4.96	6.00
9	v9	0.00	1.53	2.54	2.77	4.18	4.85	6.00
10	v10	0.00	1.41	2.59	2.86	3.93	4.53	6.00
11	v11	0.00	0.88	1.60	1.89	2.98	4.35	6.00
12	v12	0.00	0.11	1.06	1.35	2.51	4.15	6.00
13	v13	0.00	1.45	1.71	1.63	2.71	3.86	6.00
14	v14	0.00	1.55	1.68	2.59	3.54	4.36	6.00
15	v15	0.00	1.32	1.09	2.25	3.22	4.16	6.00
16	v16	0.00	1.52	1.71	2.65	3.96	4.99	6.00

An examination of the output excerpt presented shows that a number of scoring function values exhibit reversal of categories from the presumed order (such as in Items 13 and 15) or category boundaries that are very close together (such as categories 2 and 3 in Items 16). As Preston, Reise, Cai, and Hays (2011) explained, when the estimated scoring function values deviates from the presumed order of 0, 1, 2, ..., 6, the nominal model provides additional category-specific information that other ordinal polytomous IRT models (e.g., partial credit or graded) do not provide. As noted earlier, the items were scored on a 7-point scale indicating the extent to which the respondent is satisfied with the statement of the item, where: 0=terrible; 1=unhappy; 2=mostly dissatisfied; 3=mixed, about equally satisfied and dissatisfied; 4=mostly satisfied; 5=pleased; and 6=delighted. There may be some uncertainty as to how people interpret the order of the middle categories.

### 3.7.2 Bifactor Model Using the 2PL

Due to the undesirable properties when analyzing the QOL data with all 7 categories, we will again demonstrate how flexMIRT® may be used to recode data. We will fit the same bifactor factor pattern to the QOL data, but rather than utilizing all 7 categories, we will collapse the data into dichotomous responses and fit the 2PLM.

#### Example 3.10: Bifactor Structure - 2PL

```
<Project>
Title = "QOL Data";
Description = "35 Items Bifactor Showing Recoding";
<Options>
Mode = Calibration;
Quadrature = 21,5.0;
SavePRM = Yes;
Processor = 4;
```

```

<Groups>
%Group1%
File = "QOL.DAT";
Varnames = v1-v35;
N = 586;
Dimensions = 8;
Primary = 1;
Ncats(v1-v35) = 7;
Code(v1-v35) = (0,1,2,3,4,5,6),(0,0,0,1,1,1,1);
Model(v1-v35) = Graded(2);
<Constraints>
Fix (v1-v35),Slope;
Free (v1-v35),Slope(1);
Free (v2-v5),Slope(2);
Free (v6-v9),Slope(3);
Free (v10-v15),Slope(4);
Free (v16-v21),Slope(5);
Free (v22-v26),Slope(6);
Free (v27-v31),Slope(7);
Free (v32-v35),Slope(8);

```

The **Code** command is used to collapse the 7 original categories into 2. A point of note here is that the **Ncats** value provided refers to the number of categories in the raw data file, not the number of categories that will be used for modeling, which is specified with the **Model** statement. An examination of the **Code** statement shows that original responses of 0,1, and 2 will be recoded into 0s and raw responses of 3-6 will become 1s after the recoding by the program. The bifactor structure remains unchanged from the previous example, as may be seen in the in the **<Constraints>** section.

---

### Simulation

---

Simulation studies are common in the IRT literature. Compared to other commercially available IRT software programs (e.g., Bilog, IRTPro, Multilog, and Parscale), flexMIRT<sup>®</sup> is unique in that it facilitates simulations by offering the user the capability of generating data from all implemented IRT models, including multidimensional, bifactor, and multilevel models. The command syntax for generating data has the same basic structure as the model-fitting syntax. In general, it will involve either a relatively small amount of code in the syntax file, leaving the majority of the model settings and generating parameter values in the parameter file. Alternatively, one can fully specify the models and generating parameter values directly in the syntax, without using the parameter file, or using it only as a supplement. Note that a parameter file saved from a calibration run is completely compatible in simulations. This provides a convenient mechanism so that the user may conduct simulations with generating parameters found in empirical data analysis.

---

#### 4.1. A Syntax-Only Example Using Value Statement

In this example, we will use the syntax file to specify simulation of a data set consisting of 1000 simulees' responses to 4 graded items, each scored with 3 categories. The generating parameter values are specified in the **<Constraints>** section. In simulation mode, the **Value** statement can be used to specify generating parameters, as is done in the following example.

#### Example 4.1: Simulation - Graded Model Syntax Only

```
<Project>
Title = "Simulate Data";
Description = "4 Items Graded Syntax Only";
<Options>
  Mode = Simulation;
  Rndseed = 7474;
<Groups>
%G%
  File = "sim1a.dat";
  Varnames = v1-v4;
  N = 1000;
  Model(v1-v4) = Graded(3);
<Constraints>
  Value (v1),Slope,0.7;
  Value (v2),Slope,1.0;
  Value (v3),Slope,1.2;
  Value (v4),Slope,1.5;
  Value (v1,v2),Intercept(1),2.0;
  Value (v1,v2),Intercept(2),1.0;
  Value (v3,v4),Intercept(1),0;
  Value (v3,v4),Intercept(2),-1.0;
```

One can see in the `<Options>` section, flexMIRT<sup>®</sup> is set to run in Simulation mode. When this is case, a random number seed must be supplied via the `Rndseed` statement. The group specification remains the same as in calibration runs. In this case, the `File` statement instructs flexMIRT<sup>®</sup> to save the simulated item responses to “sim1a.dat”. The variables are named and the total number of simulees is set to 1000. The `Model` statement sets the generating IRT model to the graded response model with 3 categories. Next, in the `<Constraints>` section, the generating parameter values are set. For instance, the first constraint says that the value of the slope parameter for item v1 should be set to 0.7. The 5th constraint specifies that the first intercept term for item v1 and v2 should be equal to 2.0. In a single group, the group name can be omitted, so `Value (v1),Slope,0.7;` and `Value G,(v1),Slope,0.7;` would have the same effect. For multiple group simulations, the group name must be specified in each `Value` statement.

#### Output 4.1: Simulation Control Output - Graded Model Syntax Only

```
...
Summary of the Data and Control Parameters

    Missing data code      -9
    Number of Items        4
    Number of Cases        1000
    Number of Dimensions    1

Item  Categories      Model
  1         3        Graded
  2         3        Graded
  3         3        Graded
  4         3        Graded

Miscellaneous Control Values
Random number seed:      7474
...
Graded Items for Group 1: G
    Item      a      c 1      c 2
      1    0.70    2.00    1.00
      2    1.00    2.00    1.00
      3    1.20    0.00   -1.00
      4    1.50    0.00   -1.00

Graded Items for Group 1: G
    Item      a      b 1      b 2
      1    0.70   -2.86   -1.43
      2    1.00   -2.00   -1.00
      3    1.20   -0.00    0.83
      4    1.50   -0.00    0.67

...
Group Parameter Estimates:
    Group      Label      mu      s2      sd
      1         G      0.00      1.00      1.00
```

The first part of the simulation control output echoes the user-supplied specifications, (e.g. number of items, number of simulees, the number of categories, IRT model types, and the random number seed). The generating item parameters are then printed. One can see that the syntax statements are correctly interpreted. When everything is set up correctly, a simulated data set is generated and saved. The first and last three rows of this data set are shown below. The first 4 columns are the item responses (in 3 categories coded as 0,

1, and 2). Column 5 is a case ID column. Column 6 contains the generating theta value for that simulee.

**Output 4.2:** Simulated Data Set - Graded Model Syntax Only

```
0 2 1 0 0 -0.246011
2 2 2 2 1 1.70243
2 2 0 2 2 0.71107
...
2 2 1 2 997 -0.0161087
2 2 2 1 998 -0.702542
0 0 1 0 999 1.09703
```

As noted previously, simulations may either be conducted by using `Value` statements to provide generating parameter values or through the use of a parameter file. To demonstrate this point, we will now generate data with the parameters values just used, but supply them to flexMIRT® via a parameter file.

## 4.2. The First Example Redone Using Parameter File

**Example 4.2:** Simulation - Graded Model Using PRM File

```
<Project>
Title = "Simulate Data";
Description = "4 Items Graded Use PRM File";
<Options>
  Mode = Simulation;
  Rndseed = 7474;
  ReadPrmFile = "sim1b-prm.txt";
<Groups>
%G%
  File = "sim1b.dat";
  Varnames = v1-v4;
  N = 1000;
<Constraints>
```

The `ReadPrmFile` statement is invoked to read the parameter file. The group specification only contains the file name, the variable names, and the number of simulees. Note that the `<Constraints>` section is empty because the values of the generating parameters are set in the parameter file. The content of the

**Table 4.1:** Simulation - Graded Model PRM File Content

1	v2	1	1	2	3	2	1	1.0
1	v3	1	1	2	3	0	-1	1.2
1	v1	1	1	2	3	2	1	0.7
1	v4	1	1	2	3	0	-1	1.5

particular parameter file is shown in Table 4.1.

The parameter file has 4 rows, corresponding to the 4 items. Take row 1 as an example. The entry in the first column of 1 specifies that this is a row containing item parameters. This is a predefined option. The other option is 0, which corresponds to group specifications. The second column specifies the variable name. Note that the items are not ordered as v1 to v4, but there is no problem since the items are checked against the list of variable names in the syntax file and matched automatically. Column 3 indicates that this item belongs to group 1. Column 4 indicates the number of factors, in this case, the model is unidimensional. The next two columns, taken together, set the IRT model to graded (type 2 on column 5) with 3 categories (column 6). Then the item intercepts (from the intercept corresponding to the lowest category to the highest category) and the item slope are specified for each item. The resulting simulated data set is exactly the same as the previous example.

**Output 4.3:** Simulated Data Set - Graded Model Using PRM File

```
0 2 1 0 0 -0.246011
2 2 2 2 1 1.70243
2 2 0 2 2 0.71107
...
2 2 1 2 997 -0.0161087
2 2 2 1 998 -0.702542
0 0 1 0 999 1.09703
```

---

### 4.3. The Parameter File Layout

Although the progression of columns of the parameter file must proceed in a prescribed fashion, the general format of the file is plain ASCII, in which delimiters may be spaces or tabs. Column headers are not permitted, which is why the order of values in the columns must follow the rigid structure.

For arranging the values in the parameter file, we have found it easiest to use a spreadsheet program (e.g., Excel) to initially set up the entries and, when complete, export/paste the values into the flat ASCII text editor. As discussed earlier, there are set options from which the user must select for certain columns. Table 4.2 provides a summary of these options.

**Table 4.2:** Key Codes for Columns With Set Options

Column 1 (Entry Type)		Column 5 - Items (IRT Model)		Column 5 - Groups (Prior Type)		Select Columns (Nominal Contrast Type)	
Value	Meaning	Value	Meaning	Value	Meaning	Value	Meaning
0	Group	1	3PL	0	Normal Prior	0	Trend
1	Item	2	Graded	1	EH Prior	1	Identity
		3	Nominal			2	User-specified

As indicated in Table 4.2, the first column of the parameter file indicates whether the information to be supplied across the row refers to an item or a group. The information that goes into the various subsequent columns differs somewhat, depending on whether item or group is indicated in column 1, so the arrangement for these two parts of the parameter file will be discussed separately.

#### 4.3.1 Column Progression for Groups

With a 0 in the first column of the parameter file, flexMIRT<sup>®</sup> will interpret all subsequent information on the line as applicable to a group. The 2nd column is used to supply a group label, which should correspond to the label provided in the syntax file. The third column indicates to which group this group belongs, (i.e., 1 if it belongs to the first group, 2 if it belongs to the second group, etc). Column 4 requires the user to specify the number of latent variables in the generating model (within the group just specified). The 5th column is reserved for indicating the type of population distribution (prior) to be applied to the latent trait. The acceptable values are 0 for normal and 1 for empirical histogram (see Table 4.2). If a normal prior is chosen, the subsequent columns will supply the latent factor mean(s) and then (co)variance(s).

The latent factor mean values are entered in order, with the mean of factor 1 listed first, followed by mean 2, mean 3 and so on. For specifying (co)variance values, the unique elements of the latent factor variance/covariance matrix are entered in row-major order. To demonstrate this point, suppose the following  $4 \times 4$  factor covariance matrix is to be submitted to flexMIRT<sup>®</sup> :

	f1	f2	f3	f4
f1	1	2	4	7
f2	2	3	5	8
f3	4	5	6	9
f4	7	8	9	10

The progression of variance/covariance values entered in the parameter file should be

1 2 3 4 5 6 7 8 9 10

If an empirical histogram prior is chosen, the user must supply additional information. Column 6 specifies the total number of quadrature points on which the histogram is defined and Column 7 requires a maximum value for the quadrature points, which determines the range over which the points are to be distributed. Similar to specifying **Quadrature** or **FisherInf** in the **<Options>** section, the quadrature points are spread symmetrically around zero, with the minimum and maximum value determined by the entry in Column 7. After the EH specification, the mean(s) and unique element(s) in the covariance matrix are listed in the subsequent columns, following the same progression of entries just covered. The histogram itself is supplied next. For instance, if the entry in Column 6 is 101 and Column 7 is 5, then 101 points will be evenly spaced over the range of -5 to 5. The user must then supply as the last 101 entries of this particular row, the 101 ordinates of the empirical histogram, (i.e., the heights of the histogram at the 101 quadrature points). An example of simulating data with EH priors will be presented later.

#### 4.3.2 Column Progression for Items

The first four columns of a row supplying item level information have the same generic content as groups. First, a value indicating item information will follow (i.e., 1) is given in Column 1, then a variable name is assigned in Column 2. This name must correspond to a variable in the **Varnames** statement in the command file. Column 3 assigns the item to a specific group, and Column four requires the entry of the number of latent factors in the generating model. At this point (Column 5), the content of columns diverges from what was covered in the groups subsection. The generating IRT model for the item is set in Column 5, with the possible models coded as indicated in Table 4.2. For instance, an item to be generated using Graded model parameters would have a 2 in the 5th column. The subsequent column supplies the number of possible response categories for the item.

Once an IRT model is chosen, the item parameters must be entered, one value per column. The progression of expected parameters varies slightly depending on the which IRT model was selected in Column 5. If the 3PL model was selected, the guessing parameter (in logit metric) is given in Column 7, followed by the intercept parameter, and then all discrimination/slope parameter values (if there is more than one factor). If the Graded Model is selected in Column 5, the expected order for parameter values is the ordered intercept parameters (starting at the lowest in Column 7 and going up) followed by the discrimination parameter(s).

With respect to the Nominal model, the parameters are entered in the reparameterized form detailed in Thissen et al. (2010). The type of scoring contrast is given in Column 7, using one of the code values given in Table 4.2. If Trend (0) or Identity (1) contrasts are specified in Column 7, the scoring function contrast values (of which there are the number of categories minus one) must be supplied starting from Column 8. After those  $m - 1$  values are given, the slope parameter(s) are entered. Immediately after the slope parameter, the next column (the column number of which will vary depending on number of categories and slopes used) is used to indicate the contrast type for the intercept parameters. Again, if Trend or Identity contrasts are selected, the columns immediately following the contrast selection contain the appropriate number of intercept parameter values (again always equal to the number of categories minus one).

In the rare occasion that user-supplied contrasts are selected for either the scoring functions or the intercept parameters, it is necessary to supply the contrast coefficient values as well as the contrasts. In other words, the built-in contrasts are automatically computed, but any user-defined contrasts need to be entered into the program. The contrast coefficients are given after the contrast selection column (e.g., Column 7) but before the actual contrast values. The contrast coefficients of a user-supplied contrast matrix are expected to be in row major order. That is, for a contrast matrix applied to nominal items with  $k$  categories, the coefficients from that  $k \times (k - 1)$  matrix would be entered in the following order:  $T(1, 1), T(1, 2), T(1, 3), \dots, T(1, k - 1), T(2, 1), \dots, T(2, k - 1), \dots, T(k, k - 1)$ .

---

#### 4.4. A 3PL Model Simulation Example

In this example, we will simulate item responses to 10 3PL items for a single group of 500 simulees. The latent trait distribution is standard normal. The

syntax file is presented first.

**Example 4.3:** Simulation - 3PL Model Normal Population

```
<Project>
Title = "Fit Model";
Description = "10 Items 3PL N=500 Normal Priors";
<Options>
Mode = Simulation;
Progress = No;
ReadPRMFile = "genparams.txt";
RndSeed = 1;
<Groups>
%Group1%
File = "sim1.dat";
Varnames = v1-v10;
N = 500;
<Constraints>
```

As always, the command file begins with the **<Project>** section where a title and general description are provided. In the **<Options>** section we have set the engine mode to **Simulation** and, as required when **Mode = Simulation**; provided a seed value for the random number generator which will be used to create the data. Because we are not using **Value** statements, we have also provided a file name where item and group parameters may be found.

In the **<Groups>** section, we label our group and use the **File** statement to provide a name for file where the data will be stored once generated. We want to simulate data for 10 variables so we provide labels for 10 variable names with the **Varnames** statement and finally, using **N**, specify the number of cases we wish to have generated. The variable and group labels provided here must match those provided in the second column of the parameter file. As may be seen here, and as mentioned previously, the syntax command file is only used to set up the very basic structure of the analysis and the specifics are left to be input through the parameter file, to which we will now turn our attention.

As covered in Section 4.1, the actual parameter file to be submitted to flexMIRT® (Ex1genparms.txt, in this case) may not contain row headers but they are included in Table 4.3 for illustrative purposes only. In this parameter file, we have arranged all the item specifications first and the group parameters are in the bottom row. This is completely arbitrary and the rows of the file

may be arranged in any way the user wishes. However, the columns must follow the arrangement previously detailed. Reading across for the first item, the 1 in the first column denotes that the values to follow refer to an item. We then provide a variable name, `v1`, and assign this item to the first group (which is the only group in this example). The next column indicates that there is 1 latent factor in the model and the subsequent column (Column 5) says that we will be providing parameters for the 3PL model (indicated by the 1). The 6th column value tells flexMIRT<sup>®</sup> that the item will have two response options (as per the 3PL model). Based on the number of factors, the IRT model, and the number of response categories, we now supply the appropriate number of item parameters, in this case 3 parameters (1 for the lower asymptote, 1 for the intercept, and 1 for the slope). The order of parameters values must be guessing (in the logit metric), intercept (in the  $c = -a * b$  metric, where  $b$  is the difficulty/threshold), and then the slope parameter ( $a$ ).

**Table 4.3:** Simulation - 3PL Parameter File with Labeled Columns

Type	Label	Group #	# Factors	Model	# Cat	logit $g$	$c$	$a$
1	v1	1	1	1	2	-1.34	-0.69	1.30
1	v2	1	1	1	2	-0.97	-1.30	1.41
1	v3	1	1	1	2	-1.64	0.42	1.05
1	v4	1	1	1	2	-2.10	0.40	1.26
1	v5	1	1	1	2	-1.59	-1.35	2.43
1	v6	1	1	1	2	-1.72	-0.56	1.67
1	v7	1	1	1	2	-1.13	-0.26	3.12
1	v8	1	1	1	2	-1.71	-1.56	2.09
1	v9	1	1	1	2	-2.79	0.73	1.17
1	v10	1	1	1	2	-1.66	-0.14	1.10
Type	Label	Group #	# Factors	Prior Type	Mean	Var		
0	Group1	1	1	0	0.00	1.00		

The final row of the file is used to describe the group. The first 0 indicates that the row is describing a group. The second column provides the group a label, `Group1`, which should correspond to the label provided in the command file. We assign the `Group1` label to the first group by the 1 in the 3rd column, and specify that a unidimensional model will be generated, by indicating 1 factor in the 4th column. As noted in Table 4.2, the fifth column of a Group line indicates the type of prior distribution to be applied (in this case a normal prior, signified by the 0). After a normal prior is indicated, a group mean of 0.00 and a variance of 1.00 are supplied.

---

## 4.5. Simulating Data with Non-Normal Theta Distributions

In this example, we will cover data simulated from models with non-normal population distribution. This is accomplished using the empirical histograms method (previously used in Calibration Example 3.2). The empirical histogram prior may only be used for unidimensional or bifactor models (testlet included), as previously noted in the discussion of Example 3.2.

### Example 4.4: Simulation - Empirical Histogram Prior

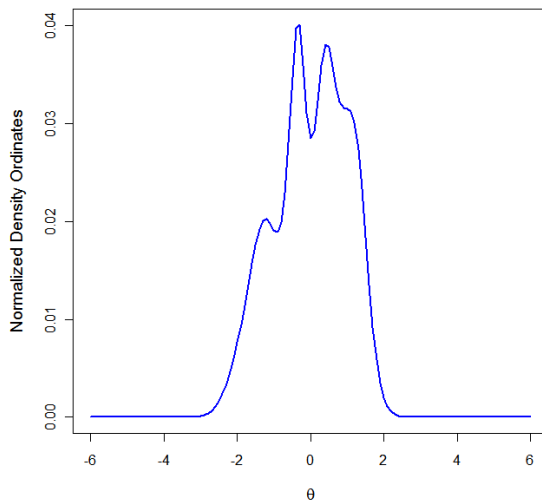
```
<Project>
Title = "Fit Model";
Description = "10 Items 3PL N=500 Non-Normal Priors";
<Options>
Mode = Simulation;
ReadPRMFile = "EHgenparams.txt";
RndSeed = 1;
<Groups>
%Group1%
File = "sim2.dat";
Varnames = v1-v10;
N = 500;
<Constraints>
```

The command file for this example is substantively the same as the previous, although we have changed the parameter file name. The added feature here is that a non-normal distribution will be specified in the generating parameter file (EHgenparams.txt). This change only effects that values entered into the Group row of the parameter file.

An examination of the generating parameter file will find that the 5th column of the Group row been changed from the value of 0, seen in Table 4.3, to a value of 1. As noted in Table 4.2, this indicates that an empirical histogram prior will be used. The 6th column then provides the number of points to be used in constructing the histogram (121) and the next value indicates the range over which the points are spread (from -6 to 6 here, as indicated by the value of 6 in the 7th column of the Group row). Following the specification of the empirical histogram parameters, the latent variable mean(s) and covariances are listed. After all means and covariances are listed,

the heights of the empirical histogram are provided. For our current example, we need to provide 121 values, per the entry in Column 6. Figure 4.1 provides a graphical display of the empirical histogram provided in the parameter file, to demonstrate the extent of non-normality.

**Figure 4.1:** Empirical Histogram From EHgenparams.txt




---

## 4.6. Simulating Bifactor Model Data

We will be using item parameters obtained from our bifactor analysis in Example 3.10 to simulate items with a bifactor structure from the 2PL model. As noted previously, when not employing **Value** statements, only the very basic structure of the analysis is supplied in the simulation syntax file and the details of the model are left to the parameter file. Table 4.4 presents a labeled version of the parameter file (QOLbifac-prm.txt) for illustrative purposes.

**Example 4.5:** Simulation - Bifactor Structure From Existing Parameter File

```
<Project>
Title = "QOL Data";
Description = "Simulation From PRM File 35 Items Bifactor";
```

```

<Options>
Mode = Simulation;
ReadPRMFile = "QOLbifac-prm.txt";
Rndseed = 7474;
<Groups>
%Group1%
File = "QOLbifacsim.DAT";
Varnames = v1-v35;
Dimensions = 8;
Primary = 1;
N = 1000;
<Constraints>

```

**Table 4.4:** Labeled Parameter File for Bifactor Model Simulation

Type	Label	Grp#	#Fac	Model	#Cat	c	$a_1$	$a_2$	$a_3$	$a_4$	$a_5$	$a_6$	$a_7$	$a_8$
1	v1	1	8	2	2	2.13	2.26	0.00	0.00	0.00	0.00	0.00	0.00	0.00
1	v2	1	8	2	2	3.78	2.31	2.51	0.00	0.00	0.00	0.00	0.00	0.00
1	v3	1	8	2	2	1.69	1.62	1.42	0.00	0.00	0.00	0.00	0.00	0.00
1	v4	1	8	2	2	3.76	3.86	4.89	0.00	0.00	0.00	0.00	0.00	0.00
1	v5	1	8	2	2	2.87	2.77	3.08	0.00	0.00	0.00	0.00	0.00	0.00
1	v6	1	8	2	2	0.86	2.77	0.00	3.20	0.00	0.00	0.00	0.00	0.00
1	v7	1	8	2	2	1.38	1.38	0.00	1.50	0.00	0.00	0.00	0.00	0.00
1	v8	1	8	2	2	0.49	2.60	0.00	2.86	0.00	0.00	0.00	0.00	0.00
1	v9	1	8	2	2	0.30	1.78	0.00	1.97	0.00	0.00	0.00	0.00	0.00
1	v10	1	8	2	2	3.09	1.91	0.00	0.00	1.74	0.00	0.00	0.00	0.00
1	v11	1	8	2	2	2.57	1.42	0.00	0.00	0.71	0.00	0.00	0.00	0.00
1	v12	1	8	2	2	1.88	1.40	0.00	0.00	0.72	0.00	0.00	0.00	0.00
1	v13	1	8	2	2	2.30	1.57	0.00	0.00	0.13	0.00	0.00	0.00	0.00
1	v14	1	8	2	2	5.16	4.02	0.00	0.00	3.80	0.00	0.00	0.00	0.00
1	v15	1	8	2	2	2.09	2.49	0.00	0.00	0.66	0.00	0.00	0.00	0.00
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
1	v34	1	8	2	2	2.95	1.67	0.00	0.00	0.00	0.00	0.00	0.00	1.30
1	v35	1	8	2	2	3.22	1.40	0.00	0.00	0.00	0.00	0.00	0.00	0.44
Type	Label	Grp#	#Fac	Prior	$\mu_1$	⋯	$\mu_8$	$\sigma_{11}$	$\sigma_{21}$	$\sigma_{22}$	$\sigma_{31}$	$\sigma_{32}$	$\sigma_{33}$	⋯
0	Group1	1	8	0	0.00	⋯	0.00	1.00	0.00	1.00	0.00	0.00	1.00	⋯

Following the parameter file format, the first column is for entering whether the values to follow on the row refer to an item or a group, with 1 denoting items and 0 denoting groups. We will now follow the first line across, discussing the columns in turn. The next column supplies a variable name for the line and the following column specifies to which group the item belongs. In this case, there is only one group, so the entire column consists of 1s. In Column 4, we specify that there are 8 factors in the model and Columns 5 and 6, respectively, tell flexMIRT<sup>®</sup> that the IRT model is the graded model (type

2) with two possible response categories. The combination of these three values informs flexMIRT<sup>®</sup> to expect 9 item parameters, one intercept and 8 slopes. Scanning down the columns of slope parameters, it becomes clear that the  $a_1$  column is for the general factor, with all item loading onto it, and the next 7 columns represent the group-specific factors, onto which only a subset of items load.

Turning now to the group specification in the bottom row of the table, the first column signifies that this line is supplying group information, due to the value of 0. Labeling, assigning the group number and the number of factors in the model, all similar to what was done in the item rows, come next. In Column 5, we enter information regarding the type of population distribution prior. We have chosen Normal priors (via the option 0). Because we have chosen Normal priors, all that is left to specify are the latent variable means for each of the eight factors, followed by the unique elements of the covariance matrix.

---

## 4.7. Simulating Group Differences

As seen in the multiple group calibration examples, each group has its own self-contained subsection within the <Groups> section. While this can lead to some redundancy in the naming of variables, etc., it does permit greater flexibility. For example, flexMIRT<sup>®</sup> does not require that the number of items, the number of categories, the kind of IRT models employed, or the number of latent dimensions to remain the same across groups. One may specify one group to have a normal prior distribution and use empirical histograms in the other group. In particular, one may even set up a multilevel model for one group and a single level model for the other.

The current example simulates group differences in both population means and variances as well as item parameters. Each group has 10 items. Group 1 has mean 0, variance 1. Group 2 has the same variance but a mean at 0.2. Group 3 has a mean of -0.2 and variance of 1.5. The items are for the most part 3PL items, with the exception of Item 1 in Group 3. The lower asymptote parameter is set to 0 for that item, making it a 2PL item. Also, the intercept parameter for Item 9 in Group 2 is chosen such that its threshold value is 0.5 standard deviation higher than the corresponding items in Groups 1 and 3.

#### Example 4.6: Simulation - DIF and Group Mean Differences

```
<Project>
Title = "Simulate Data";
Description = "3 Groups 10 Items N=1000 (mostly) 3PLs
Group 1 N( 0.0,1.0), Group 2 N( 0.2,1.0), Group 3 N(-0.2,1.5)
Item 9 in Group 2 has 0.5 higher threshold than the corresponding
item in Groups 1 and 3.
Item 1 in Group 3 is actually a 2PL item.";
<Options>
Mode = Simulation;
ReadPRMFile = "genparams.txt";
RndSeed = 7474;
<Groups>
%Group1%
File = "group1.dat";
Varnames = v1-v10;
N = 1000;
%Group2%
File = "group2.dat";
Varnames = v1-v10;
N = 1000;
%Group3%
File = "group3.dat";
Varnames = v1-v10;
N = 1000;
<Constraints>
```

The <Options> section remains substantially unchanged from previous examples. Within the <Groups> section, we now have labels and specifications for the three distinct groups for which we wish to generate data. Following the other examples, the details of the simulation are set up in the parameter file (genparams.txt). The contents of the parameter file (presented, in part, in Table 4.5) are now discussed.

As before, the first column still indicates whether the row refers to an item or a group, and the second column provides a label. The first indication that this is a multiple group problem appears in Column 3, which is the column designated for assigning the row (item or group) to a specific group defined in the syntax. In previous examples, with only one group, all rows displayed a 1

**Table 4.5:** Simulating DIF and Group Mean Differences - Parameter Values

1	v1	1	1	1	2	-1.33972	-0.68782	1.301362
1	v2	1	1	1	2	-0.96814	-1.29641	1.409536
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
1	v9	1	1	1	2	-2.79306	0.730972	1.172468
1	v10	1	1	1	2	-1.65629	-0.14263	1.102673
1	v1	2	1	1	2	-1.33972	-0.68782	1.301362
1	v2	2	1	1	2	-0.96814	-1.29641	1.409536
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
1	v9	2	1	1	2	-2.79306	0.144738	1.172468
1	v10	2	1	1	2	-1.65629	-0.14263	1.102673
1	v1	3	1	2	2	-0.68782	1.301362	
1	v2	3	1	1	2	-0.96814	-1.29641	1.409536
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
1	v9	3	1	1	2	-2.79306	0.730972	1.172468
1	v10	3	1	1	2	-1.65629	-0.14263	1.102673
0	Group1	1	1	0	0	1		
0	Group2	2	1	0	0.2	1		
0	Group3	3	1	0	-0.2	1.5		

in this column, where now the first 10 items (4 of which are displayed in the table) have 1s, the next 10 rows have 2s and so on.

With respect to the differences across the groups, as noted earlier, Item 1 in group 3 is a 2PL item, so it is specified as a graded model with two response categories, while item 1 in both Groups 1 and 2 are 3PL items. DIF also occurs for Item 9. The same generating IRT model is used and the guessing and discrimination parameter values are identical across groups, but in Group 2, the difficulty parameter value is 0.5 higher (in the  $b$  metric) than the value in Groups 1 and 3. Mean differences among the groups are also generated. The last 3 rows refer to Groups 1 through 3, respectively. After labeling, specifying group membership, and declaring the number of factors, normal population distributions are chosen for all groups (via the 0s in the 5 column). The sixth column provides the population means and the seventh column provides population variances.

---

## 4.8. Multilevel Bifactor Model Simulation

Our final example will illustrate how one may simulate item responses according to a multilevel bifactor model in two groups. We will generate data for 12 dichotomously scored graded items.

### Example 4.7: Simulation - Multilevel Model with Bifactor Structure

```
<Project>
Title = "Two-level Bifactor Model (Two Groups)";
Description = "12 Items, 4 Group Factors,
100 L2 units, 10 respondents within each";

<Options>
Mode = Simulation;
Rndseed = 7471;
ReadPrmFile = "MGsimL2-prm.txt";

<Groups>
%G1%
File = "simL2.g1.dat";
VarNames = v1-v12;
Dimensions = 6;
Between = 1;
N = 1000;
Nlevel2 = 100;

%G2%
File = "simL2.g2.dat";
VarNames = v1-v12;
Dimensions = 6;
Between = 1;
N = 1000;
Nlevel2 = 100;

<Constraints>
```

Skipping down to the `<Groups>` section and focusing on the first group's subsection, we have provided the data file name, variable labels, and the total number of dimensions of the generating model with the `Dimensions` state-

ment. The command `Between= 1;` indicates that the model has one level-2 (between) latent variable and 5 level-1 (within) latent variables. The `Nlevel2` command is required when simulating multilevel data and determines the number of level-2 units (e.g., schools in which students are nested). In this case, the number of level-2 units is equal to 100. In conjunction with the total number of level-1 cases specified in the `N=1000` line, one can deduce that the number of level-1 units within each level-2 unit is equal to 10. flexMIRT<sup>®</sup> will distribute the total sample size as evenly as possible across the level-2 units. If the sample size value is not a multiple of the number of level-2 units, the “remainder” observations will be added to the final level-2 unit. Note that while the specifications for the second group are the same as those in the first group in this example, it is by no means a requirement. As mentioned earlier, the number of items, type of IRT model, and the latent dimensionality can be different across groups.

#### Output 4.4: Simulation Control Output for Multilevel Bifactor Model

Two-level Bifactor Model (Two Groups)							
12 Items, 4 Group Factors, 100 L2 units, 10 respondents within each							
Summary of the Data and Control Parameters							
Group	G1	G2					
Missing data code	-9	-9					
Number of Items	12	12					
Number of L-2 Units	100	100					
Number of L-1 Units	1000	1000					
Number of Dimensions	6	6					
Between Dimensions	1	1					
Within Dimensions	5	5					
...							
Miscellaneous Control Values							
Random number seed:	7471						
...							
2PL Items for Group 1: G1							
Item	a 1	a 2	a 3	a 4	a 5	a 6	c
1	2.00	2.00	1.50	0.00	0.00	0.00	-1.50
2	1.50	1.50	1.50	0.00	0.00	0.00	-0.50
3	1.00	1.00	1.20	0.00	0.00	0.00	0.50
4	2.00	2.00	0.00	1.20	0.00	0.00	1.50
5	1.50	1.50	0.00	1.50	0.00	0.00	-1.50
6	1.00	1.00	0.00	1.80	0.00	0.00	-0.50
7	1.50	1.50	0.00	0.00	1.20	0.00	0.50
8	1.50	1.50	0.00	0.00	1.50	0.00	1.50
9	1.00	1.00	0.00	0.00	1.50	0.00	-1.50
10	2.00	2.00	0.00	0.00	0.00	1.80	-0.50
11	2.00	2.00	0.00	0.00	0.00	2.00	0.50
12	1.00	1.00	0.00	0.00	0.00	1.50	1.50
...							
Group Latent Variable Means:							

Group	Label	mu 1	mu 2	mu 3	mu 4	mu 5	mu 6
1	G1	0.00	0.00	0.00	0.00	0.00	0.00
2	G2	0.20	0.00	0.00	0.00	0.00	0.00

Latent Variable Variance-Covariance Matrix for Group1: G1						
Theta 1	Theta 2	Theta 3	Theta 4	Theta 5	Theta 6	
0.50						
0.00	1.00					
0.00	0.00	1.00				
0.00	0.00	0.00	1.00			
0.00	0.00	0.00	0.00	1.00		
0.00	0.00	0.00	0.00	0.00	1.00	

Latent Variable Variance-Covariance Matrix for Group2: G2						
Theta 1	Theta 2	Theta 3	Theta 4	Theta 5	Theta 6	
0.25						
0.00	1.00					
0.00	0.00	1.00				
0.00	0.00	0.00	1.00			
0.00	0.00	0.00	0.00	1.00		
0.00	0.00	0.00	0.00	0.00	1.00	

The simulation control output shows that the input syntax and the parameter values are specified correctly. The item parameters are invariant across groups so only parameters from Group 1 are shown. The within-item equality of the slopes implies that the 5 level-1 dimensions (a2 - a6) follow a testlet response theory model (restricted bifactor) with 4 testlets of 3 items each. Furthermore, the equality of slopes on the Within general dimension (a2) and the Between general dimension (a1) implies that a random-intercept model is set up that provides a decomposition of the Between and Within variances for the general trait. The mean of Group 1 and 0.20 lower than Group 2, but the Group 1 Between variance is 0.50 and the Group 2 between variance is 0.25.

---

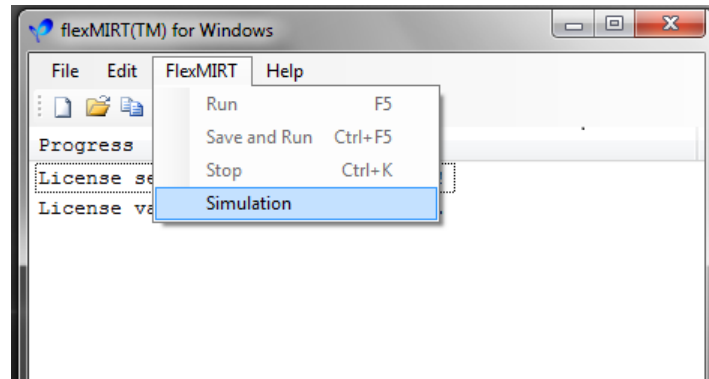
## 4.9. Internal Simulation Function

Here we provide an overview for using the internal simulation function (ISF), which allows users to automate numerous replications of a given simulation and subsequent calibration. This function uses existing simulation and calibration syntax files and merely provides a convenient way of obtaining replications; it is not an alternative method for constructing simulation syntax files.

### 4.9.1 Accessing the Internal Simulation Function

Currently, the ISF is available through the flexMIRT<sup>®</sup> GUI only; specifically, the ISF is not available if using the program through the command prompt or when it is being called by another program, such as R. The ISF is found under the “flexMIRT” menu bar option of the flexMIRT<sup>®</sup> GUI.

**Figure 4.2:** Accessing the ISF through the GUI



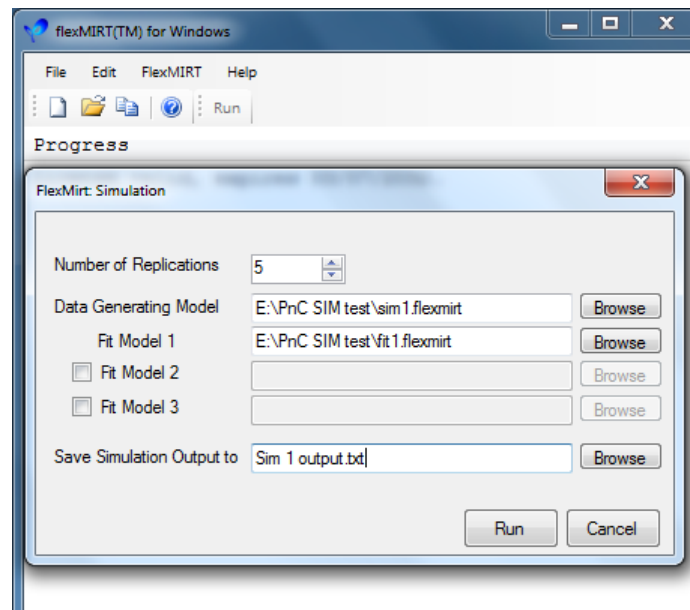
Once “Simulation” is selected, a pop-up will appear where the user enters the information for the simulation study to be conducted. As can be seen in the next image, the user specifies the desired number of replications, the name and location of the existing simulation syntax file which will be responsible for the data creation, and the location(s) and name(s) of the calibration syntax for the model(s) that the users wishes to fit to the simulated data.

Currently, there is no way to save the simulated data but in an upcoming update, flexMIRT<sup>®</sup> will output the simulated data sets to external files by default. The simulated datasets will be output to the file name specified in the simulation syntax file but will have the replication number added to the output file name. For instance, if we are outputting the data to the file “SIM.dat” via the `File = "SIM.dat";` command in the simulation syntax, the

individual replications will be saved to files “SIM-0.dat,” “SIM-1.dat,” etc.. However, in the calibration syntax, to direct flexMIRT<sup>®</sup> to the datafile, we will still use the statement `File = SIM.dat;` - without any mention of the replication number.

When all the syntax files have been identified and a name for the output file is specified in the “Save Simulation Output to:” blank, the “Run” button will be enabled.

**Figure 4.3:** Completed ISF pop-up window



Once the simulation has been submitted, flexMIRT<sup>®</sup> will begin the process of simulating data and fitting the specified model(s) to the data. The progress of each replication is printed in the engine viewing pane, as with any one-off simulation or calibration. When the specified number of replications have completed, a single output file is created. The format of the ISF output file is optimized to be read into some other software program (e.g., SAS) and as such, appears quite different from the normal flexMIRT<sup>®</sup> output. The output file has no labeled sections or column headers, etc.. Due to this, the layout of the ISF output file will be described in detail.

### 4.9.2 The Simulation Output File

The first 6 columns in the output for every model summary will contain the same information. Column 1 has the simulation replication number. Column 2 prints what is called the return code and is an indicator of successful replications. If the return code is 0, the replication completed successfully; if it takes any value other than 0, a problem was encountered. Column 3 reports the log likelihood, Column 4 gives the number of estimation cycles that were completed to fit the model, the 5th column reports the time the calibration run took, and the 6th column give the inverse of the condition number. The inverse condition number is equal to the ratio of the smallest to largest eigenvalue of the asymptotic covariance matrix of the item parameters; if it is very small, it shows instability of the obtained solution.

**Output 4.5:** Beginning Columns of Output from 5 Replications

1	0	-4.42522e+003	46	1.41000e-001	8.61952e-003	2.87409e+000	1.99261e+000	7.62795e-001
2	0	-4.44206e+003	40	1.25000e-001	1.52923e-002	3.00412e+000	1.88355e+000	7.38170e-001
3	0	-4.28512e+003	41	1.25000e-001	8.18986e-003	2.78522e+000	1.87415e+000	7.44810e-001
4	0	-4.33888e+003	31	1.09000e-001	1.63897e-002	2.74887e+000	1.86261e+000	6.77367e-001
5	0	-4.37110e+003	29	1.09000e-001	1.82420e-002	2.58602e+000	1.84299e+000	8.12819e-001

At the seventh column, reporting of the item and latent variables parameters begins. The parameters are reported in the same order as they are numbered in a corresponding one-off calibration run output. It is strongly recommended that a single run of the calibration syntax file be conducted prior to employing the ISF, to both ensure the syntax works properly and to obtain the ordering of parameters. For example, below is the item parameter portion of the flexMIRT<sup>®</sup> output for a single run fitting the model found in the file “Fit1.flexmirt”.

**Output 4.6:** Item Parameters with Parameter Number Labels

Graded Items for Group 1: G1											
Item	Label	P#	a	s.e.	P#	c 1	s.e.	P#	c 2	s.e.	
1	v1	3	0.81	0.13	1	2.59	0.14	2	1.84	0.11	
2	v2	6	1.14	0.15	4	2.69	0.16	5	2.01	0.13	
3	v3	9	1.41	0.15	7	2.92	0.18	8	0.20	0.09	
4	v4	12	1.25	0.13	10	2.88	0.16	11	0.13	0.08	
5	v5	15	2.13	0.28	13	2.76	0.24	14	1.70	0.18	
6	v6	18	0.54	0.09	16	2.92	0.14	17	0.20	0.07	

The order in which the parameters will be printed in the ISF output cor-

responds to the values list in the columns labeled “P#” in the output. From this output excerpt, we can see that in the ISF output file, the two intercept parameters and then the slope parameter for Item 1 will be printed first, as indicated by the 1, 2, and 3, respectively, in the “P#” column. These values are followed by the intercept parameters and slope for Item 2, and so on. Once the item parameters have been reported, the latent variable mean(s) and covariance(s) are printed. After all point estimates are given, the respective standard error for each parameter is printed, in the same order as the point estimates.

The next six columns of the ISF output, following all parameter point estimates and SEs, are fixed. flexMIRT<sup>®</sup> will report, in order, the  $G^2$ ,  $X^2$ , and  $M_2$  overall fit statistics. It should be noted here that the  $M_2$  column will always print, regardless of whether the statistic was requested or not. If the statistic was not requested, via the GOF and M2 statements in the <Options> section, placeholder values of 0.0000 will be printed and are not indicative of the actual value of the  $M_2$  statistic, had it been requested.

The overall fit statistics are followed by three members of the power-divergence family of fit statistics (e.g., Cressie & Read, 1984) for assessing latent distribution fit, specifically the summed score likelihood based  $G^2$ ,  $D^2$ , and  $X^2$ . These indices have been found to accurately detect departures from normality within the latent distribution while appropriately “ignoring” other forms of model misspecification such as multidimensionality (Li & Cai, 2012).

#### Output 4.7: ISF output - Switch from Model 1 to Model 2 Values

...	1.29309e-314	1.47651e-317		1	0	-4.52136E+03	31	...
...	9.70097e-315	1.47651e-317		2	0	-4.40664E+03	24	...
...	2.32371e-315	1.47651e-317		3	0	-4.44256E+03	24	...
...	1.04469e-314	1.47651e-317		4	0	-4.45391E+03	21	...
...	1.02812e-314	1.47651e-317		5	0	-4.58027E+03	22	...

If the ISF was instructed to fit multiple models to the data, then the values for next model will follow, on the same line, after the latent distribution fit statistics of the current model. The new model reporting begins with the replication number and follows the same general order of reporting detailed previously. If we fit two models to a given set of simulated data, the output columns at the point where the reporting for the first model ends and the second model begins will look something like the values presented in Output

4.7 (although we have added the break line for emphasis). Note, however, that the numbering of parameters may change from model to model. As stated before, to determine the reporting order of parameters, a single run of any model being fit is strongly recommended before using the ISF.

---

### Metropolis-Hastings Robbins-Monro Estimation for High Dimensional Models

---

Although Bock and Aitkin’s (1981) EM algorithm made IRT parameter estimation practical, the method does have shortcomings with the most noticeable one being its limited ability to generalize to truly high-dimensional models. This is due primarily to the need to evaluate high-dimensional integrals in the likelihood function for the item parameters. As the number of dimensions of a model increases linearly, the number of quadrature points increases exponentially, making EM estimation unwieldy and computationally expensive for models with more than a handful of latent factors.

A Metropolis-Hastings Robbins-Monro (MH-RM; Cai, 2010b, 2010c) algorithm has been implemented in flexMIRT<sup>®</sup> to allow for the estimation of higher dimensional models. We will briefly discuss the algorithm to acquaint those users less familiar with this recently introduced method, cover the available flexMIRT<sup>®</sup> options and syntax commands that are specific to this estimation routine, and then provide working examples.

---

#### 5.1. Overview of MH-RM Estimation

In brief, the MH-RM algorithm is a data augmented Robbins-Monro type (RM; Robbins & Monro, 1951) stochastic approximation (SA) algorithm driven by the random imputations produced by a Metropolis-Hastings sampler (MH; Hastings, 1970; Metropolis, Rosenbluth, Rosenbluth, Teller, & Teller, 1953), which is a sampling method based on the principles of Markov chain Monte Carlo (MCMC). The MH-RM algorithm is motivated by Tittering’s (1984) recursive algorithm for incomplete data estimation, and is a close relative of Gu and Kong’s (1998) SA algorithm. It can also be conceived of as an extension of the Stochastic Approximation EM algorithm (SAEM; Celeux & Diebolt, 1991; Celeux, Chauveau, & Diebolt, 1995; Delyon, Lavielle, & Moulines, 1999) when

linear sufficient statistics do not naturally exist in the complete data model.

Within the flexMIRT<sup>®</sup> implementation, the MH-RM iterations use initial values found through an unweighted least squares factor extraction stage (Stage I) and further improved upon during a supplemented EM-like stage (Stage II) - both Stage I and Stage II use a fixed number of cycles. Stage III is home to the MH-RM algorithm and where the primary estimation of parameters occurs. Cycles in Stage III terminate when the estimates stabilize.

Cycle  $j + 1$  of the MH-RM algorithm for multidimensional IRT consists of three steps:

- *Imputation.* In the first steps, conditional on provisional item and latent density parameter estimates  $\boldsymbol{\beta}^{(j)}$  from the previous cycle, random samples of the individual latent traits  $\boldsymbol{\theta}^{(j+1)}$  are imputed using the MH sampler from a Markov chain having the posterior of the individual latent traits  $\pi(\boldsymbol{\theta}|\mathbf{Y}, \boldsymbol{\beta}^{(j)})$  as the unique invariant distribution.
- *Approximation.* In the second step, based on the imputed data, the complete data log-likelihood and its derivatives are evaluated so that the ascent directions for the item and latent density parameters can be determined later.
- *Robbins-Monro Update.* In the third step, RM stochastic approximation filters are applied when updating the estimates of item and latent density parameters. First, the RM filter will be applied to obtain a recursive stochastic approximation of the conditional expectation of the complete data information matrix. Next, we use the RM filter again when updating the new parameter estimates. Cai (2010c) showed that the sequence of parameters converges with probability 1 to a local maximum of the likelihood of the parameter estimates, given the observed data.

For more detailed descriptions of the estimation method, see Cai (2010b, 2010c).

---

## 5.2. MH-RM Specific Syntax Options

**Example 5.1:** <Options> - Engine Mode Settings to Call MH-RM Estimation and MH-RM Specific Options

```
<Options >  
Mode = Calibration;  
Algorithm = MHRM;  
RndSeed = ?;  
Imputations = 1;  
Thinning = 10;  
Burnin = 10;  
Stage1 = 200;  
Stage2 = 100;  
Stage3 = 2000;  
Stage4 = 0;  
InitGain = 1.0;  
Alpha = 1.0;  
Epsilon = 1.0;  
WindowSize = 3;  
ProposalStd = 1.0;  
ProposalStd2 = 1.0;  
MCsize = 25000;
```

To call the MH-RM algorithm, the combination of **Mode = Calibration;**, **Algorithm = MHRM;**, and a random number seed via **RndSeed = ;** is required. All other listed commands are optional, with the default values shown above.

**Imputations** controls the number of imputations from the MH step per RM cycle. A larger value will lead to smoother behavior from the algorithm. In most cases, a value of 1 should be sufficient for obtaining accurate point estimations, however this number may be increased for better standard errors.

Thinning refers to the MCMC practice of retaining only every  $k$ th draw from a chain. Thinning is used, in part, to reduce the possible autocorrelation that may exist between adjacent draws. The **Thinning** statement sets the interval for the MH sampler - meaning if **Thinning = 10;**, every 10th draw by the sampler will be retained.

The **Burnin** statement controls the number of draws that are discarded from the start of the MH sampler. These draws are discarded to avoid using

values that were sampled before the chain had fully converged to the target distribution. `Burnin = 10;` tells flexMIRT<sup>®</sup> to discard the first 10 values obtained by the MH sampler.

`Stage1` determines the number of Stage I (constant gain) cycles. The Stage I iterations are used to improve default or user-supplied starting values for the estimation that occurs in Stage II. `Stage2` specifies the number of Stage II (Stochastic EM, constant gain) cycles. The Stage II iterations are used to further improve starting values for the MH-RM estimation that occurs in Stage III. `Stage3` sets the maximum number of allowed MH-RM cycles to be performed.

`Stage4` determines the method by which SEs are found. If `Stage4 = 0;` then SEs will be approximated recursively (see Cai, 2010b). If a non-zero value is given then the Louis formula (Louis, 1982) is used directly. If the Louis formula is to be used, the supplied value determines the number of iterations of the SE estimation routine; this number will typically need to be large (e.g., 1000 or more).

`InitGain` is the gain constant for Stage I and Stage II. If the algorithm is initially taking steps that are too large this value may be reduced from the default to force smaller steps.

`Alpha` and `Epsilon` are both Stage III decay speed tuning constants. `Alpha` is the first tuning constant and is analogous to the `InitGain` used in Stages I and II. `Epsilon` controls the rate at which the sequence of gain constants converge to zero. Specified `Epsilon` values must be in the range (0.5, 1], with a value closer to 1 indicating a faster rate of convergence to zero. `WindowSize` allows the user to set the convergence monitor window size. Convergence of the MH-RM algorithm is monitored by calculating a window of successive differences in parameter estimates, with the iterations terminating only when all differences in the window are less than 0.0001. The default value of the window size is set at 3 to prevent premature stoppage due to random variation.

`ProposalStd` and `ProposalStd2` control the dispersion of the Metropolis proposal densities for the first and second level of the specified model, respectively. If a single level model is specified, only the `ProposalStd` command will be used. **Although default values of 1.0 have been set for both of these commands, these values need to be adjusted on a case-by-case basis.** The values used will depend on the complexity of the model, the number of items, the type of items (e.g., dichotomous, polytomous), and the model fit to the items (e.g., graded, 3PLM, Nominal). The user should choose `ProposalStd` and `ProposalStd2` values so that the long-term average of the

acceptance rates (which is printed for each iteration in the progress window) for level 1 and level 2 (if applicable) are around 0.5 for lower dimensional models ( $\leq 3$  factors) and in the 0.2 - 0.3 range for higher dimensional/more complex models. Generally speaking, increasing the **ProposalStd** value will result in lowered acceptance rates while decreasing the value will result in higher acceptance rates. Users are directed to Roberts and Rosenthal (2001) for optimal scaling, choice of dispersion constants, and long-term acceptance rates of Metropolis samplers.

**MCSIZE** is the Monte Carlo size for final log-likelihood, AIC, and BIC approximations.

---

### 5.3. Examples Utilizing MH-RM Estimation

All of the calibration examples presented previously in the manual are able to be estimated using MH-RM estimation. However, requests for additional things such as LD indices, more complete GOF statistics, etc. will most likely be ignored by flexMIRT<sup>®</sup> because their behavior, in conjunction with the MH-RM algorithm, is still under research.

#### 5.3.1 MIRT Model with Correlated Factors

For a first example, we will refit the 4 correlated factors model applied to the 35-item QOL dataset, originally presented in Example 3.8.

##### Example 5.2: MHRM: 4 Factor MIRT Model

```
<Project>
Title = "QOL Data";
Description = "Items 2-21 Correlated Factors";

<Options>
  Mode = Calibration;

  Algorithm=MHRM;
  RndSeed = 432;
  ProposalStd = 0.5;

  SavePRM = Yes;
  Processor = 4;
  FactorLoadings = Yes;
```

```

<Groups>

%Group1%
  File = "QOL.DAT";
  Varnames = v1-v35;
  Select = v1-v21;

  N = 586;

  Dimensions = 4;
  Ncats(v1-v21) = 7;
  Model(v1-v21) = Graded(7);

<Constraints>
  Fix (v1-v21),Slope;

  Free (v1,v2-v5),Slope(1);
  Free (v1,v6-v9),Slope(2);
  Free (v1,v10-v15),Slope(3);
  Free (v1,v16-v21),Slope(4);

  Free Cov(2,1);

  Free Cov(3,1);
  Free Cov(3,2);

  Free Cov(4,1);
  Free Cov(4,2);
  Free Cov(4,3);

```

In the <Options> section, we have specified that flexMIRT<sup>®</sup> should use the MH-RM algorithm via `Algorithm=MHRM`;. In addition, we have provided the required random number seed for the MH-RM estimation and set `ProposalStd = 0.5`;. This value was adjusted from the default of 1.0 because a first run of the algorithm was observed to have an acceptance rate lower than is desirable. As noted earlier, lowering the `ProposalStd` value will generally lead to higher acceptance rates. Outside of the <Options> section, no syntax was changed from the example presented in Chapter 3; the same model was fit by both the

MH-RM and EM estimation routines.

**Output 5.1:** Correlated Four Factor Model Output - Processing Pane Output

MH-RM: Stage I									
#	1;	AR:	0.45,0.00;	Fn:	-23418.96				
#	2;	AR:	0.46,0.00;	Fn:	-21948.41				
#	3;	AR:	0.39,0.00;	Fn:	-21098.19				
#	4;	AR:	0.35,0.00;	Fn:	-20510.71				
#	5;	AR:	0.30,0.00;	Fn:	-20335.52				
#	6;	AR:	0.27,0.00;	Fn:	-20145.06				
#	7;	AR:	0.27,0.00;	Fn:	-20118.94				
#	8;	AR:	0.25,0.00;	Fn:	-20052.04				
#	9;	AR:	0.24,0.00;	Fn:	-20065.11				
#	10;	AR:	0.23,0.00;	Fn:	-20092.89				
#	11;	AR:	0.24,0.00;	Fn:	-20071.26				
#	12;	AR:	0.23,0.00;	Fn:	-20086.42				
#	13;	AR:	0.24,0.00;	Fn:	-20087.42				
...									
MH-RM: Stage II									
#	1;	AR:	0.19,0.00;	Fn:	-20033.45				
#	2;	AR:	0.19,0.00;	Fn:	-19997.80				
#	3;	AR:	0.18,0.00;	Fn:	-20009.39				
#	4;	AR:	0.19,0.00;	Fn:	-20004.81				
#	5;	AR:	0.20,0.00;	Fn:	-20043.30				
...									
MH-RM: Stage III									
#	1;	AR:	0.19,0.00;	Fn:	-20055.31;	Gam:	0.50;	W:	0; P#: 12; Chg: 0.2178
#	2;	AR:	0.19,0.00;	Fn:	-20112.81;	Gam:	0.33;	W:	0; P#: 0; Chg: 0.0777
#	3;	AR:	0.20,0.00;	Fn:	-20042.63;	Gam:	0.31;	W:	0; P#: 12; Chg: 0.0943
...									

As noted previously, acceptance rates are printed in the processing pane for each iteration at all three stages of the MH-RM estimation. Given the complex structure of the model and the IRT model applied to the items, the desired goal for our fitted model is to have the reported acceptance rate have an average value between 0.2 and 0.3. Reading across the output, flexMIRT® is reporting the iteration number, the letters “AR” for acceptance rate, followed by the level-1 acceptance and then the level-2 acceptance rate. The current model had only 1 level, so all level-2 values will be 0.00. The final value, labeled Fn, is an indicator that the program is or is not working towards a converged solution. While not the log likelihood, it may be interpreted in a similar fashion, meaning that less negative values indicate a better fit. Examining the first several rows of the provided iteration history, the acceptance rate starts somewhat high but quickly drops down to the near desired level by the 10th iteration of Stage I; during this time, the Fn value also rapidly drops initially

and then stabilizes. For all of Stages II and III, both the acceptance rate and the function value fluctuate, with the acceptance rate staying near the desired value of 0.20. While the output should be closely examined for issues, this pattern of values in the processing pane is an early indicator that the MH-RM algorithm is working towards a good solution.

We present the same section of the output was that given for Example 3.8.

**Output 5.2:** Correlated Four Factor Model Output - Loadings, Means and Variances

Factor Loadings for Group 1: Group1												
Item	Label	lambda 1	s.e.	lambda 2	s.e.	lambda 3	s.e.	lambda 4	s.e.			
1	v2	0.83	0.03	0.00	----	0.00	----	0.00	----			
2	v3	0.78	0.03	0.00	----	0.00	----	0.00	----			
3	v4	0.93	0.01	0.00	----	0.00	----	0.00	----			
4	v5	0.92	0.01	0.00	----	0.00	----	0.00	----			
5	v6	0.00	----	0.87	0.02	0.00	----	0.00	----			
6	v7	0.00	----	0.72	0.04	0.00	----	0.00	----			
7	v8	0.00	----	0.90	0.02	0.00	----	0.00	----			
8	v9	0.00	----	0.88	0.02	0.00	----	0.00	----			
9	v10	0.00	----	0.00	----	0.66	0.05	0.00	----			
10	v11	0.00	----	0.00	----	0.74	0.04	0.00	----			
11	v12	0.00	----	0.00	----	0.71	0.04	0.00	----			
12	v13	0.00	----	0.00	----	0.72	0.04	0.00	----			
13	v14	0.00	----	0.00	----	0.77	0.04	0.00	----			
14	v15	0.00	----	0.00	----	0.72	0.04	0.00	----			
15	v16	0.00	----	0.00	----	0.00	----	0.77	0.03			
16	v17	0.00	----	0.00	----	0.00	----	0.73	0.04			
17	v18	0.00	----	0.00	----	0.00	----	0.76	0.04			
18	v19	0.00	----	0.00	----	0.00	----	0.84	0.03			
19	v20	0.00	----	0.00	----	0.00	----	0.75	0.04			
20	v21	0.00	----	0.00	----	0.00	----	0.49	0.06			

QOL Data  
Items 2-21 Correlated Factors

Group Parameter Estimates:

Group	Label	P#	mu 1	s.e.	P#	mu 2	s.e.	P#	mu 3	s.e.	P#	mu 4	s.e.
1	Group1		0.00	----		0.00	----		0.00	----		0.00	----

Latent Variable Variance-Covariance Matrix for Group1: Group1

P#	Theta 1	s.e.	P#	Theta 2	s.e.	P#	Theta 3	s.e.	P#	Theta 4	s.e.
	1.00	----									
141	0.40	0.03		1.00	----						
142	0.54	0.03	143	0.49	0.03		1.00	----			
144	0.54	0.03	145	0.55	0.03	146	0.68	0.02		1.00	----

From the excerpt of the output provided, it may again be observed that the output from the MH-RM is formatted identically to output from EM estimations. Comparing across the two estimation routine results, the parameter estimates are nearly identical indicating that the EM and MH-RM estimation routines are reaching a similar converged solution. In favor of the MH-RM, however, is the fact that the total time needed to estimate the model and generate output was around 4 minutes, compared to the 54 minutes used by the default Bock and Aitkin EM.

### 5.3.2 Multi-level MIRT Model

With the four factor example, we have demonstrated the time benefit that can be obtained through the use of the flexMIRT<sup>®</sup> MH-RM algorithm when estimating high dimensional models. The MH-RM as implemented in flexMIRT<sup>®</sup> is also able to fit multi-level models, a feature that, to our knowledge, is not available in any other commercially available software. We will provide syntax for and output from fitting such a model. The data used in this example was simulated to have six dichotomous items with responses from 2000 individuals—these 2000 individuals are nested within 100 higher-order units, 20 per unit. This type of structure could be seen if 2000 students from 100 different schools took a brief assessment in which items were graded as right/wrong.

#### Example 5.3: MHRM: Two-level MIRT Model

```
<Project>
Title = "Fit Two-level MIRT Model to Simulated Data";
Description = "6 Items, 100 L2 units, 20 respondents within each";

<Options>
  Mode = Calibration;
  Rndseed = 10;
  Algorithm = MHRM;
  ProposalStd = 1.0;
  ProposalStd2 = 1.0;
  Processors = 2;
  MCsize = 10000;

<Groups>

%Gr%
  File = "simL2.dat";
  Varnames = v1-v6,l2id;
  Select = v1-v6;
  Cluster = l2id;
  Dimensions = 4;
  Between = 2;
  N = 2000;
  Ncats(v1-v6) = 2;
  Model(v1-v6) = Graded(2);

<Constraints>
  Fix (v1-v6),Slope; // fix all slopes to begin with
```

```

Free (v1-v3),Slope(1); // level-2 factor 1
Free (v4-v6),Slope(2); // level-2 factor 2
Free (v1-v3),Slope(3); // level-1 factor 1
Free (v4-v6),Slope(4); // level-1 factor 2

Equal Gr,(v1-v3),Slope(1):Gr,(v1-v3),Slope(3); // cross-level equality
Equal Gr,(v4-v6),Slope(2):Gr,(v4-v6),Slope(4);

Free Cov(1,1);
Free Cov(2,2);
Free Cov(1,2);
Free Cov(3,4);

```

As before, we have told flexMIRT<sup>®</sup> to use the MH-RM algorithm and provided a random number seed. In this example, we are using both the `ProposalStd` and `ProposalStd2` keywords because we now have two levels to our model. The adjustments to the `ProposalStd` will affect the level-1 acceptance rate and adjustments to the `ProposalStd2` value will affect the level-2 acceptance rates. In the `<Groups>` section we inform flexMIRT<sup>®</sup> we will be fitting a model with 4 latent dimensions (`Dimensions = 4;`), two of which will be used as higher-level “between” factors (`Between = 2;`), and we then indicate a variable that will supply the level-2 group membership (`Cluster = l2id;`). Within the `<Constraints>` section, we create a structure that has 2 “between” and 2 “within” factors (via the first group of `Fix` and `Free` statements) and assign items to factors as desired. We also constrain the slopes across the levels to equality using `Equal` statements and then allow the variances of the level-2 factors and the covariance of the within factors and the covariance of the between factors to be freely estimated. It is worth noting here that there is nothing outside of the `<Options>` section that is uniquely related to the requested MH-RM estimation - this specific model can be estimated using the default Bock-Aitkin EM, albeit with noticeable time loss. Larger models with more factors would most likely require MH-RM to complete successfully.

We will cover the processing pane output prior to addressing the results of the estimation. In particular, we will highlight the differences between the previous example and the processing pane output for a model with a second level.

### Output 5.3: Two-Level MIRT Model - Processing Pane Output

```
MH-RM: Stage I
# 1; AR: 0.47,0.63; Fn: -11738.27
# 2; AR: 0.48,0.61; Fn: -11666.05
# 3; AR: 0.47,0.59; Fn: -11556.98
# 4; AR: 0.46,0.59; Fn: -11447.69
# 5; AR: 0.45,0.60; Fn: -11366.38
# 6; AR: 0.44,0.57; Fn: -11236.31
# 7; AR: 0.44,0.60; Fn: -11164.65
# 8; AR: 0.43,0.58; Fn: -11131.30
# 9; AR: 0.42,0.61; Fn: -11147.14
# 10; AR: 0.43,0.60; Fn: -11092.40
# 11; AR: 0.43,0.51; Fn: -10987.70
...
MH-RM: Stage II
# 1; AR: 0.41,0.54; Fn: -10993.18
# 2; AR: 0.41,0.59; Fn: -10991.12
# 3; AR: 0.42,0.58; Fn: -11074.56
# 4; AR: 0.42,0.60; Fn: -11010.83
# 5; AR: 0.41,0.61; Fn: -11032.11
...
MH-RM: Stage III
# 1; AR: 0.41,0.56; Fn: -10954.38; Gam: 0.50; W: 0; P#: 4; Chg: 0.0451
# 2; AR: 0.41,0.56; Fn: -10846.63; Gam: 0.33; W: 0; P#: 15; Chg: 0.0011
# 3; AR: 0.41,0.55; Fn: -10934.18; Gam: 0.29; W: 0; P#: 15; Chg: -0.0013
# 4; AR: 0.41,0.54; Fn: -10915.08; Gam: 0.25; W: 0; P#: 12; Chg: 0.0199
```

While the general structure of the reported values is the same as the previous processing pane, rather than “0.00” being consistently repeated as the second value after the **AR:** as in the first example, there is now a value that changes with each iteration - this is the acceptance rate for the second level. To be concrete, in Iteration 1 of Stage I, the reported level-1 acceptance rate is 0.47 and the level-2 acceptance rate is 0.63. As noted before, the desired range of acceptance rates is between 0.20 and 0.30 for large or complex models, with values around 0.50 being acceptable for smaller, less dimensionally complex models. Output 5.4 presents the parameters for the estimated model. It may again be observed that the item and group parameter output from the MH-RM is formatted identically to output from EM estimations. Additionally, it appears that the estimation completed successfully, resulting in reasonable item and group parameter values and SEs.

## Output 5.4: Multilevel MIRT Model - Item and Group Parameters

Fit Two-level MIRT Model to Simulated Data																
6 Items, 100 L2 units, 20 respondents within each																
2PL Items for Group 1: Gr																
Item	Label	P#	a 1	s.e.	P#	a 2	s.e.	P#	a 3	s.e.	P#	a 4	s.e.	P#	c	s.e.
1	v1	7	1.59	0.09		0.00	----	7	1.59	0.09		0.00	----	1	-1.04	0.08
2	v2	8	1.82	0.11		0.00	----	8	1.82	0.11		0.00	----	2	-0.52	0.07
3	v3	9	0.97	0.06		0.00	----	9	0.97	0.06		0.00	----	3	0.68	0.06
4	v4		0.00	----	10	1.79	0.11		0.00	----	10	1.79	0.11	4	1.64	0.10
5	v5		0.00	----	11	1.62	0.10		0.00	----	11	1.62	0.10	5	-1.82	0.10
6	v6		0.00	----	12	0.89	0.05		0.00	----	12	0.89	0.05	6	-0.35	0.05
Fit Two-level MIRT Model to Simulated Data																
6 Items, 100 L2 units, 20 respondents within each																
Group Parameter Estimates:																
Group	Label	P#	mu 1	s.e.	P#	mu 2	s.e.	P#	mu 3	s.e.	P#	mu 4	s.e.			
1	Gr		0.00	----		0.00	----		0.00	----		0.00	----			
Latent Variable Variance-Covariance Matrix for Group1: Gr																
P# Theta 1	s.e.	P# Theta 2	s.e.	P# Theta 3	s.e.	P# Theta 4	s.e.									
13	0.74	0.12														
15	0.47	0.10	14	0.88	0.14											
	0.00	----		0.00	----											
	0.00	----				16	0.47	0.02	1.00	----						

---

## 5.4. Fixed Effects Calibration

New to flexMIRT<sup>®</sup> is the ability to perform fixed effects (aka fixed theta) calibration, in which the user supplies individual theta values, which are fixed, to be used in calibrating items. This mode of calibration is potentially useful in field-testing experimental items. Fixed effects calibration is only available in conjunction with MH-RM estimation. The keyword to induce flexMIRT<sup>®</sup> to perform fixed effects calibration is **FixedTheta**. **FixedTheta** is a group-specific command that is used to set the variable(s) that contain(s) the known, individual theta values for that group in the calibration. By implementing fixed effects calibration in flexMIRT<sup>®</sup> as group-specific, it is possible to construct a calibration in which one group is based on fixed theta values and the other group is based on random, estimated thetas.

### Example 5.4: Fixed Effects Calibration

```
<Project>
Title = "LSAT 6 - Fixed Effects Calibration";
Description = "5 Items 1PL N=1000";

<Options>
  Mode = Calibration;
  Algorithm = MHRM;
  Stage1 = 0;
  Stage2 = 0;

<Groups>
%Group1%
  File = "lsat6.dat";
  Varnames = v1-v5,id,map;
  Select = v1-v5;
  N = 1000;
  NCats(v1-v5) = 2;
  Model(v1-v5) = Graded(2);
  FixedTheta = map;

<Constraints>
  Equal (v1-v5),Slope;
```

For any group that will be calibrated with fixed ability values, theta is read-in as part of the datafile for that group. In this example, the theta values for %Group1% are in the variable *map*, the last variable listed in the **Varnames =** ; statement. As in earlier examples, because we have provided variables that are not to be calibrated, we then use a **Select** statement to specify only those items which will be subjected to IRT analysis. To provide flexMIRT® with the theta values, we use the statement **FixedTheta = map;** to tell the program the variable that holds the fixed theta values. Although not seen in this example, **FixedTheta** works under multidimensionality - currently, the number of variables listed in **FixedTheta = vars;** must match the number of factors specified via **Dimensions** for that group. Of final note in the fixed effects calibration syntax, the **Stage1** and **Stage2** keywords in the <Options> section, which determine the number of constant gain and stochastic EM, constant gain cycles of the MH-RM estimation method, are both set to 0. This is recommended for fixed effects calibration in flexMIRT® because these cycles do not contribute much in the fixed effects case and eliminating them improves the time-efficiency of the estimation.

When **FixedTheta** is specified and the program can successfully run a problem, flexMIRT® will print in the main output a notification that fixed effects calibration was performed, prior to reporting the item parameter estimates for the group. Such a notification is presented in the example program output. Also with regard to the output, please note that for a single-group fixed effects analysis or a fixed effects analysis that applies to all the groups in a multiple-group run, the SE of the log-likelihood approximation will be reported as 0.

### Output 5.5: Fixed Effects Calibration Output - Item Parameters

*** Fixed effects calibration in Group 1: Group1									
LSAT 6									
5 Items 1PL N=1000									
2PL Items for Group 1: Group1									
Item	Label	P#	a	s.e.	P#	c	s.e.	b	s.e.
1	v1	6	2.96	0.08	1	3.59	0.14	-1.21	0.03
2	v2	6	2.96	0.08	2	1.35	0.08	-0.45	0.01
3	v3	6	2.96	0.08	3	0.33	0.08	-0.11	0.00
4	v4	6	2.96	0.08	4	1.76	0.09	-0.59	0.02
5	v5	6	2.96	0.08	5	2.79	0.11	-0.94	0.03

---

### Cognitive Diagnostic Models

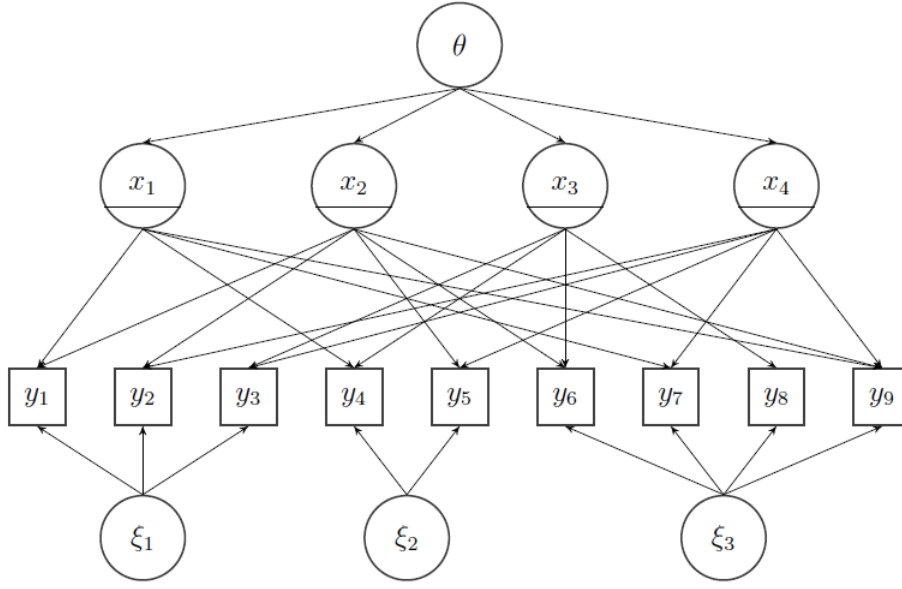
---

Over the last several decades, a growing field of interest and research has been the development and estimation of latent variable models that locate respondents in multidimensional space for the purpose of diagnosis; in this context, “diagnosis” includes the usual meaning, such as diagnosing a psychological disorder, but also encompasses activities such as identifying a student’s mastery of content areas in an educational setting. Such models have been variously termed cognitive diagnostic models (CDMs), diagnostic classification models, cognitive psychometric models, restricted latent class models, and structured IRT models, among others. The goal of this chapter is not to provide a didactic on CDMs (interested readers are directed to Rupp, Templin, and Henson (2010) for a thorough discussion of CDMs and related models) but to demonstrate, for those already familiar with the models, how flexMIRT<sup>®</sup> may be used to fit them.

## 6.1. General CDM Modeling Framework

Complete coverage of the models flexMIRT<sup>®</sup> fits to CDMs is provided in the Models chapter, but we will briefly introduce the models here. Figure 6.1 provides a schematic for a generic flexMIRT<sup>®</sup> CDM with a single higher-order factor to facilitate the discussion of the mathematical equations.

**Figure 6.1:** Generic Higher-order CDM



The extended CDMs used by flexMIRT<sup>®</sup> combine a linear predictor with a link function. The linear predictor may be represented as:

$$\begin{aligned} \eta = & \alpha + \sum_{k_1=1}^K \beta_{k_1} x_{k_1} + \sum_{k_1=1}^K \sum_{k_2=1}^{(k_1-1)} \beta_{k_1, k_2} x_{k_1} x_{k_2} \\ & + \text{higher-order interaction terms} + \sum_{k=1}^p \lambda_k \xi_k \end{aligned} \quad (6.1)$$

where  $\alpha$ ,  $\beta$ , and  $\lambda$  are item parameters,  $x$ s are 0/1 attributes, and  $\xi$ s are continuous latent variables. The kernel (linear predictor)  $\eta$  can be turned into item response probabilities through the use of appropriate link functions, such

as the logit presented below.

$$P(y_i = 1|\mathbf{x}, \xi) = \pi_i(1|\mathbf{x}, \xi) = \frac{1}{1 + \exp[-(\eta)]}$$

For the higher-order portion of the CDM, flexMIRT<sup>®</sup> is modeling the  $x$  probabilities, (i.e.,  $P(x_1, \dots, x_K|\theta_1, \dots, \theta_q) = \prod_{k=1}^K P(x_k|\theta_1, \dots, \theta_q)$ ) with a set of (higher-order) latent variables ( $\theta$ ), so that each of the attributes,  $xs$ , is treated as if it is an observed item.

## 6.2. CDM Specific Syntax Options

**Example 6.1:** <Options>, <Groups>, and <Constraints>- CDM Specific Options

```

<Options >
DMtable = Yes/No;

<Groups >
Attributes = ?;
InteractionEffects = (?, ?, ...);
Generate = (?, ?), (?, ?), ...;

DM = group;

<Constraints >
Coeff group, (vars), parameter, ?;

Fix group, (vars), MainEffect;
Free group, (vars), Interaction(?, ?, ..)

```

In the <Options> section, the user may control whether or not the (sometimes large) diagnostic classification probability table is printed in the output via the **DMtable** command.

**Attributes** is used to set the number of main effects present in the skill space that is being assessed. This is the keyword that triggers flexMIRT<sup>®</sup> to model discrete latent variables, rather than the default continuous latent variables.

**InteractionEffects** instructs flexMIRT<sup>®</sup> to automatically generate all possible interaction effects. For instance, with 4 attributes one may specify

**InteractionEffects** = (2,3,4); and, in addition to the 4 main effects, the program will also generate the 2nd, 3rd, and 4th order interaction terms of the attributes as dimensions of the model. If only the second order interaction effects are desired, those would be created by specifying **InteractionEffects** = (2);.

Similar to the **InteractionEffects** keyword, **Generate** sets up higher order interactions effects. However, rather than creating all possible interaction effects **Generate** creates only those effects specified in the statement. For example, **Generate** = (3,6,7),(4,7); is used to generate the interaction effect of attributes 3, 6, and 7 and, separately, the interaction effect of attributes 4 and 7. There is no limit to the number of interaction effects that may be specified in a **Generate** statement and the order in which interaction effects are specified does not matter.

The **DM** command stands for “diagnostic model” and is used to set the group containing observed data - this is the group to which the higher-order DM will be applied. The **DM** command is optional; one could, in principle, fit a model without the DM (higher-order) latent variables.

In the <**Constraints**> section, the **Coeff** command allows user’s to impose proportionality restrictions on the item parameters (i.e., **MainEffects** and **Interactions**). For instance, a DINO model may be parameterized as a logistic model with interactions but the interaction coefficients are -1.0 times the main effect slopes. With the **Coeff** keyword, we can change the sign of those specific interaction terms, effectively enabling flexMIRT® to fit more CDMs than it would be capable of fitting if restricted to only the **MainEffect** and **Interaction** keywords.

The **Fix** and **Free** commands noted in the <**Constraints**> section aren’t limited to those particular commands or to CDMs, but are provided to show that rather than using the typical IRT parameter keywords (Slope, etc.), users may employ the parameter keywords **MainEffect** and **Interaction** to allow syntax specification that is in keeping with the CDM conceptualization/terminology for the models.

---

### 6.3. CDM Modeling Examples

In this section, we will provide several examples of CDMs fit in flexMIRT®. We will discuss basic examples that demonstrate how to fit several of the more well-known CDMs that appear in the literature and then, using datasets that have been fit elsewhere, demonstrate the comparability of flexMIRT® estimates to those from other programs and estimation methods.

### 6.3.1 Introductory CDM Examples

Here we provide straight-forward demonstrations of several widely discussed models. We hope that in addition to providing simple, concrete examples for the chosen models, from these examples users will begin to see the flexibility flexMIRT<sup>®</sup> has in fitting CDMs, gain insight into the details of our syntax for CDMs, and be able to extrapolate this to fit the wide variety of alternative CDMs that are not specifically covered but may be fit using flexMIRT<sup>®</sup>. For the examples in this section, we will use simulated data for 15 dichotomous items that were generated to measure four underlying attributes. The Q-matrix, which assigns items to attributes, is presented in Table 6.1. As can be seen, the majority of items use only 1 attribute, but the final 3 items are assigned to both attributes 3 and 4. These items could be fit with a variety of models and we will present examples for three different CDMs.

**Table 6.1:** Q-matrix for Basic CDM Demonstrations

Item	Attribute 1	Attribute 2	Attribute 3	Attribute 4
Item 1	1	0	0	0
Item 2	1	0	0	0
Item 3	1	0	0	0
Item 4	0	1	0	0
Item 5	0	1	0	0
Item 6	0	1	0	0
Item 7	0	0	1	0
Item 8	0	0	1	0
Item 9	0	0	1	0
Item 10	0	0	0	1
Item 11	0	0	0	1
Item 12	0	0	0	1
Item 13	0	0	1	1
Item 14	0	0	1	1
Item 15	0	0	1	1

#### Basic C-RUM Fit to Simulated Data

For the first example, the majority of items will only depend on the main effect of a single attribute and we will fit the compensatory reparameterized unified model (C-RUM) to the final three items. The C-RUM model (e.g., Hartz,

2002; Choi, Rupp, & Pan, 2013) is a CDM that, when items are informed by multiple attributes, allows for mastery of one attribute to compensate for non-mastery on other attributes. The C-RUM estimates for each item an intercept term and as many slope terms for an item as there are “1” entries in the Q-matrix. As detailed in Choi et al. (2013), C-RUM is a special case of the log-linear cognitive diagnosis model (LCDM) in which all interaction terms are set to 0, meaning only main effects are estimated.

**Example 6.2:** C-RUM Model Fit To Simulated Data

```
<Project>
Title = "Calibrate and Score DM Data";
Description = "C-RUM Model";

<Options>
  Mode = Calibration;
  MaxE = 20000;
  MaxM = 5;
  Mtol = 0;
  Etol = 1e-5;
  SE = REM;
  SavePrm = Yes;
  SaveCov = Yes;
  SaveSco = Yes;
  Score = EAP;

<Groups>

%%
File = "DMsim.dat";
Varnames = v1-v15;

N = 3000;

Ncats(v1-v15) = 2;
Model(v1-v15) = Graded(2);

Attributes = 4;
Dimensions = 4;
```

```

%D%
  Varnames = a1-a4;
  DM = G;

<Constraints>
  Fix G,(v1-v15),MainEffect;

  Free G,(v1-v3),MainEffect(1);
  Free G,(v4-v6),MainEffect(2);
  Free G,(v7-v9, v13-v15),MainEffect(3);
  Free G,(v10-v15),MainEffect(4);

```

The first points of note in the syntax occur in the <Options> section. The maximum number of E-steps has been increased from the default, the E tolerance and M tolerance values have been decreased, and the SE calculation method has been set to REM (Richardson extrapolation method), as experience has found the REM SEs to be preferable to estimates from other methods. These changes in the <Options> section are made to improve the resulting SE estimates and are recommended for any CDMs fit using flexMIRT®.

The specifications for the CDM begin in the <Groups> section. Although the naming of the groups is arbitrary, all examples will use the custom of defining group G as the group containing observed data and group D as the group containing the higher-order latent dimensions/attributes. As with a typical IRT model, in group G we read in our data from a named file, provide flexMIRT® with the variable names, and specify the sample size. The items were simulated as correct/incorrect items, so Ncat= has been set to 2 and we will fit the 2PLM to all items. For the first statement specific to the CDM, we tell flexMIRT® that 4 latent attributes will be used. Because the C-RUM model uses only main effect terms, the total number of dimensions is equal to the number of attributes.

In the D group, we inform flexMIRT® that the higher-order portion of the model we are setting up will be applied to the group with observed data, group G, using the DM = G; statement. We then name the four attributes via Varnames = a1- a4;. If no other information is given regarding categories or a model to be fit, flexMIRT® by default will assume there are two categories for each of the attributes and will fit the attributes with the 2PLM. In the <Constraints> section, we assign items onto attributes according to the Q-matrix. First, we fix all items to 0 and then free items onto the appropriate main effect and interaction terms. Items 1 - 12 only have main effects for a

single attribute specified in the <Constraints> section. From the Q-matrix, we know that items 13 - 15 will load on both attributes 3 and 4. These specifications are incorporated in the statements `Free G, (v7-v9, v13-v15), MainEffect(3);` and `Free G, (v10-v15), MainEffect(4);`, in which items 13 - 15 appear in the list of items freed onto both attribute 3 and attribute 4.

**Output 6.1: C-RUM Output - Item Parameters and Attribute Profile Posterior Probabilities**

Item	Label	P#	a	1	s.e.	P#	a	2	s.e.	P#	a	3	s.e.	P#	a	4	s.e.	P#	c	s.e.
1	v1	24	1.89	0.14	----		0.00	0.00	----		0.00	0.00	----	1	----	0.00	----	1	-1.68	0.07
2	v2	25	1.91	0.15	----		0.00	0.00	----		0.00	0.00	----	2	----	0.00	----	2	-1.14	0.07
3	v3	26	1.91	0.17	----		0.00	0.00	----		0.00	0.00	----	3	----	0.00	----	3	-0.63	0.06
4	v4		0.00	----	27	2.43	0.25	0.17	0.25	----		0.00	----	4	----	0.00	----	4	-0.34	0.07
5	v5		0.00	----	28	2.23	0.17	0.00	0.17	----		0.00	----	5	----	0.00	----	5	-1.16	0.08
6	v6		0.00	----	29	1.88	0.33	0.00	0.33	----		0.00	----	6	----	0.00	----	6	1.54	0.07
7	v7		0.00	----		0.00	----	0.00	----	30	1.93	0.14	0.14	7	----	0.00	----	7	-1.29	0.10
8	v8		0.00	----		0.00	----	0.00	----	31	2.16	0.15	0.15	8	----	0.00	----	8	-0.85	0.09
9	v9		0.00	----		0.00	----	0.00	----	32	1.69	0.13	0.13	9	----	0.00	----	9	-0.37	0.08
10	v10		0.00	----		0.00	----	0.00	----		0.00	----	----	36	2.47	0.18	0.18	10	-0.05	0.11
11	v11		0.00	----		0.00	----	0.00	----		0.00	----	----	37	1.68	0.17	0.17	11	0.77	0.11
12	v12		0.00	----		0.00	----	0.00	----		0.00	----	----	38	1.92	0.20	0.20	12	1.14	0.12
13	v13		0.00	----		0.00	----	0.00	----	33	0.10	0.41	0.41	39	0.96	0.42	0.42	13	2.77	0.22
14	v14		0.00	----		0.00	----	0.00	----	34	0.70	0.28	0.28	40	1.03	0.26	0.26	14	1.60	0.14
15	v15		0.00	----		0.00	----	0.00	----	35	0.55	0.15	0.15	41	1.85	0.20	0.20	15	-1.38	0.13

Diagnostic IRT Attributes and Cross-classification Probabilities for Group 1: G

Pattern	Prob
0 0 0	0.15002832
0 0 1	0.16623281
0 0 1 0	0.03889021
0 0 1 1	0.19192117
0 1 0 0	0.00750342
0 1 0 1	0.03906408
0 1 1 0	0.00531547
0 1 1 1	0.11005979
1 0 0 0	0.00145117
1 0 0 1	0.01593558
1 0 1 0	0.00165291
1 0 1 1	0.07127746
1 1 0 0	0.00033627
1 1 0 1	0.01527851
1 1 1 0	0.00093495
1 1 1 1	0.18411789

The first section of the output excerpt presents the item parameters estimates, with freely-estimated versus fixed-at-zero parameters mirroring the Q-matrix specifications. The factors labeled a1-a4 represent main effects/slope parameters for attributes 1 - 4, respectively, and c is the intercept parameter.

The next section of interest is labeled **Diagnostic IRT Attributes and Cross-classification Probabilities for Group 1: G**, which reports the estimated proportion of the population in each attribute profile. For instance, it is expected that about 15% of the population have mastered none of the measured attributes (attribute profile [0 0 0 0]), around 4 % have mastered attributes 2 and 4 (profile [0 1 0 1]), and 18.4% of the population have mastered all 4 attributes (profile [1 1 1 1]).

We will next cover the structure of the individual scoring file (-sco) that was requested in the <Options> section. In the -sco output excerpt presented, we have given only the first 10 cases and have included line breaks (//) and a separator (|) not in the output file for ease of presentation.

Output 6.2: C-RUM Output -SCO Output

The first two columns of the -sco file give the group and observation number, similar to a standard flexMIRT<sup>®</sup> IRT score output file. Recall that for this example, there are 4 attributes and 16 possible attribute profiles. The estimated probability of the respondent having mastered each of the attributes individually is given, starting after the observation number. For example, for observation 1, the probability they have mastery of attribute 1 is 0.054, for attribute 2 it is 0.128 and for attribute 3 it is 0.165 and for attribute 4 it is 0.375. The following four columns, after the separator, are the estimated standard errors for those probabilities, respectively.

The entries that have been moved to the second line of each observation contain the estimated attribute posterior variance/covariance matrix. For observation 1, those values would enter the matrix as:

	a1	a2	a3	a4
a1	0.050755			
a2	0.023774	0.109961		
a3	0.024396	0.033344	0.137477	
a4	0.029047	0.049426	0.051856	0.234379

In what we have made the third line for each respondent are the posterior probabilities for each of the possible attribute profiles. As noted earlier with 4 attributes, there are 16 possible profiles and, therefore, 16 values on the third line for this example. For observation 1, the posterior probability of being in attribute profile 1 is 0.548, the probability of possessing attribute profile 2 is 0.204 and so on. The order in which the attribute profile probabilities are given matches the order of the probability profiles used in the output table “Diagnostic IRT Attributes and Cross-classification Probabilities.” Looking back at the output presented for this example, we find that attribute profile 1 corresponds to [0 0 0 0] (meaning no attributes are mastered) and attribute profile 2 corresponds to [0 0 0 1] (meaning only attribute 4 has been mastered), etc.

Finally, in what has been moved to the fourth line for each respondent the most likely attribute profile is listed, followed by the individual’s estimated theta score for the higher-order latent variable, as well as the the SD and variance estimates for the higher-order theta estimate. For respondent 1, the most likely attribute profile is profile 1, corresponding to attribute profile [0 0 0 0] (mastery of no attributes); this attribute can be found to have the highest posterior probability listed on the line above.

## Basic DINA Fit to Simulated Data

For the second example, we will fit the deterministic input, noisy-and-gate (DINA) model to the final three items. Unlike the C-RUM, this model is non-compensatory in nature, not allowing mastery on one attribute to make up for lack of mastery on other attributes. This is accomplished in the LCDM framework by constraining main effects to zero and allowing only the most complex attribute interaction term for an item to take on a non-zero value.

### Example 6.3: DINA Fit to Simulated Data

```
<Project>
Title = "Calibrate and Score DM Data";
Description = "DINA Model";

<Options>
  Mode = Calibration;
  MaxE = 20000;
  MaxM = 5;
  Etol = 1e-5;
  SE = REM;
  SavePrm = Yes;
  SaveCov = Yes;
  SaveSco = Yes;
  Score = SSC;

<Groups>

%G%
  File = "DMsim.dat";
  Varnames = v1-v15;

  N = 3000;

  Ncats(v1-v15) = 2;
  Model(v1-v15) = Graded(2);

  Attributes = 4;
  InteractionEffects = (2);
  Dimensions = 10; //4 ME and 6 2nd order ints.

%D%
  Varnames = a1-a4;
  DM = G;
```

```

<Constraints>
  Fix G,(v1-v15),MainEffect;

  Free G,(v1-v3),MainEffect(1);
  Free G,(v4-v6),MainEffect(2);
  Free G,(v7-v9),MainEffect(3);
  Free G,(v10-v12),MainEffect(4);

  // item 13-15 fit with DINA model that depends on int of attribs 3,4
  Free G,(v13-v15),Interaction(3,4);

```

All syntax in the <Options> section remains unchanged from the previous example, with the exception of requesting sum score to EAP conversion tables (via `Score = SSC;`). In the `G` group, we have declared the data file, variable names, sample size, number of categories and IRT model to be fit to each item, as in the previous example. We again specify that there are 4 attributes with main effects, but because we are using the DINA model we will also need to construct dimensions that will represent the interaction effects. This is accomplished via `InteractionEffects = (2);` which is requesting that all second order interaction terms (i.e., attribute 1 with attribute 2, attribute 1 with attribute 3, etc.) be created for the model. Because we have requested all second order interactions (of which there are 6 possible with the 4 attributes), the total number of dimensions for the final model is 10 (4 main effect terms and 6 second order interactions). Syntax in the `D` group remains unchanged from the previous example. In the <Constraints> section, as with the previous example, we initially fix all items to zero and then free items onto the appropriate attributes. Unlike the previous example, which used only main effects, we are also using the parameter keyword `Interaction` to free items 13-15 onto the interaction of attributes 3 and 4. As noted when introducing the model, for items that depend on more than one attribute, only the highest order interaction terms is allowed to be non-zero, so we do not free items 13-15 onto the main effects of attributes 3 or 4.

Because the majority of the output has the same structure as the previous example's, regardless of specific CDM fit to items, we will only briefly discuss outputted estimates. For this example, we highlight the output that relates to the item parameters and the sum score to EAP conversion table in the -ssc file that is printed due to `Score = SSC;`.

**Output 6.3:** DINA output - Item Parameter Estimates

2PL Items for Group 1: G														
Item	Label	P#	a 1	s.e.	P#	a 2	s.e.	P#	a 3	s.e.	P#	a 4	s.e.	P#
1	v1	24	1.87	0.14		0.00	----		0.00	----		0.00	----	
2	v2	25	1.93	0.15		0.00	----		0.00	----		0.00	----	
3	v3	26	1.90	0.17		0.00	----		0.00	----		0.00	----	
4	v4		0.00	----	27	2.38	0.25		0.00	----		0.00	----	
5	v5		0.00	----	28	2.23	0.18		0.00	----		0.00	----	
6	v6		0.00	----	29	1.87	0.32		0.00	----		0.00	----	
7	v7		0.00	----		0.00	----	30	2.01	0.15		0.00	----	
8	v8		0.00	----		0.00	----	31	2.16	0.15		0.00	----	
9	v9		0.00	----		0.00	----	32	1.63	0.13		0.00	----	
10	v10		0.00	----		0.00	----		0.00	----	33	2.53	0.17	
11	v11		0.00	----		0.00	----		0.00	----	34	1.70	0.18	
12	v12		0.00	----		0.00	----		0.00	----	35	1.90	0.21	
13	v13		0.00	----		0.00	----		0.00	----		0.00	----	
14	v14		0.00	----		0.00	----		0.00	----		0.00	----	
15	v15		0.00	----		0.00	----		0.00	----		0.00	----	

Item	Label	P#	a 7	s.e.	P#	a 8	s.e.	P#	a 9	s.e.	P#	a 10	s.e.	P#
1	v1		0.00	----		0.00	----		0.00	----	1	-1.67	0.07	
2	v2		0.00	----		0.00	----		0.00	----	2	-1.14	0.07	
3	v3		0.00	----		0.00	----		0.00	----	3	-0.63	0.06	
4	v4		0.00	----		0.00	----		0.00	----	4	-0.34	0.07	
5	v5		0.00	----		0.00	----		0.00	----	5	-1.17	0.08	
6	v6		0.00	----		0.00	----		0.00	----	6	1.54	0.07	
7	v7		0.00	----		0.00	----		0.00	----	7	-1.55	0.13	
8	v8		0.00	----		0.00	----		0.00	----	8	-1.09	0.11	
9	v9		0.00	----		0.00	----		0.00	----	9	-0.52	0.09	
10	v10		0.00	----		0.00	----		0.00	----	10	0.04	0.09	
11	v11		0.00	----		0.00	----		0.00	----	11	0.83	0.11	
12	v12		0.00	----		0.00	----		0.00	----	12	1.24	0.11	
13	v13		0.00	----		0.00	----		0.00	----	36	0.74	0.30	
14	v14		0.00	----		0.00	----		0.00	----	37	1.33	0.22	
15	v15		0.00	----		0.00	----		0.00	----	38	1.58	0.12	

The output excerpt presents the estimated parameters for the items, with estimated versus fixed-at-zero loadings mirroring the Q-matrix specifications. The factors labeled **a1** - **a4** represent main effects for attributes 1 - 4 and **a5** - **a10** represent the requested 2nd order interactions. Items 1 - 12 have estimates for main effects on only the attributes specified in the Q-matrix and item 13-15 have estimates on only the interaction of attributes 3 and 4, which is labeled as **a10** in the parameter table. One may notice that dimensions **a5** - **a9** that have no items assigned to them - these dimensions correspond to the unused second order interaction effects that were created via the **InteractionEffects** keyword. The estimated item values reported are in the LCDM parameterization, but the classic DINA parameters of guessing and slippage may be obtained by conversion. Guessing is found from the reported flexMIRT<sup>®</sup> parameters as  $\exp(c)/(1 + \exp(c))$  and one minus the slippage term  $(1 - s)$  is found as  $\exp(c + a)/(1 + \exp(c + a))$ .

In the -ssc file that was generated by the scoring request, we find tables that provide conversions from sum scores to the probability of mastery for each attribute. We present an excerpt of these table below.

**Output 6.4:** DINA Summed Score Conversion Tables from -ssc File

Summed Score to Attribute Profile Probability Conversion Table:												
Summed Score	EAP 1	EAP 2	EAP 3	EAP 4	SD 1	SD 2	SD 3	SD 4	P	Error Covariance Matrix		
0.00	0.001	0.001	0.026	0.005	0.030	0.026	0.158	0.070	0.0000002	0.000876		
										0.000003	0.000700	
										0.000115	0.000045	0.024902
										0.000017	0.000007	-0.000092
												0.004948
1.00	0.001	0.001	0.029	0.010	0.032	0.035	0.168	0.097	0.0000093	0.001025		
										0.000006	0.001256	
										0.000147	0.000085	0.028217
										0.000035	0.000017	-0.000168
												0.009475
2.00	0.001	0.002	0.036	0.021	0.036	0.049	0.186	0.143	0.0001508	0.001332		
										0.000013	0.002420	
										0.000214	0.000173	0.034470
										0.000084	0.000041	-0.000363
												0.020508
...												
13.00	0.760	0.795	0.990	0.983	0.427	0.404	0.097	0.129	0.0805989	0.182430		
										0.027666	0.163028	
										0.002884	0.002254	0.009474
										0.005347	0.003948	0.001326
												0.016527
14.00	0.898	0.889	0.997	0.992	0.303	0.315	0.053	0.087	0.0356330	0.091595		
										0.013076	0.099016	
										0.000808	0.000623	0.002789
										0.001972	0.001464	0.000291
												0.007550
15.00	0.966	0.947	0.999	0.996	0.180	0.225	0.027	0.060	0.0078999	0.032539		
										0.004564	0.050540	
										0.000175	0.000147	0.000739
										0.000572	0.000493	0.000057
												0.003578

For this example, each summed score is listed with four EAP values, corresponding to the 4 attributes specified in the model, 4 SD values, the ex-

pected proportion of respondents who will obtain the listed summed score (listed in the column labeled **p**), and the error variance/covariance matrix associated with that summed score. For example, given a summed score of 0, the probability that the individual has mastered attribute 1 is 0.001, the probability that they have mastered attribute 3 is 0.026 and so on. For individuals with a summed score of 15, those probabilities increase to 0.966 and 0.999, respectively. Rather than simply providing the most likely attribute profile associated with a summed score, flexMIRT® prints the individual attribute probabilities so researchers have sufficient information to apply mastery/indifference/non-mastery cut-off values appropriate for the specific assessment at hand.

### Basic DINO Fit to Simulated Data

The final basic example using this simulated dataset will fit the disjunctive, deterministic noisy-or-gate (DINO) model. Unlike the previous models, the DINO allows for both non-zero main effect and interaction terms. However, and again using the LDCM framework, constraints are placed, such that the main effect slopes are set equal and the interaction term is -1 times the main effect constrained-to-equality slope. This is done because an item may be endorsed/answered correctly if any one of the attributes is mastered and there is assumed to be no added benefit to having mastery of several/all attributes over mastering only one or a subset of the attributes - the -1 applied to the interaction term serves to remove such a “benefit” from the model.

#### Example 6.4: DINO Fit to Simulated Data

```
<Project>
Title = "Calibrate and Score Simulated DM Data";
Description = "DINO Model";

<Options>
  Mode = Calibration;
  MaxE = 20000;
  Etol = 1e-5;
  SE = REM;
  SavePrm = Yes;
  SaveCov = Yes;
  SaveSco = Yes;
  Score = EAP;

<Groups>

%G%
File = "DMsim.dat";
Varnames = v1-v15;
```

```

N = 3000;

Ncats(v1-v15) = 2;
Model(v1-v15) = Graded(2);

Attributes = 4;
Generate = (3,4);
Dimensions = 5; //4 ME and 1 2nd order int.

%D%
Varnames = a1-a4;
DM = G;

<Constraints>
Fix G,(v1-v15),MainEffect;

Free G,(v1-v3),MainEffect(1);
Free G,(v4-v6),MainEffect(2);
Free G,(v7-v9),MainEffect(3);
Free G,(v10-v12),MainEffect(4);

// items 13-15 fit with DINO model that depend on int of attribs 3,4
Free G,(v13-v15),MainEffect(3);
Free G,(v13-v15),MainEffect(4);
Free G,(v13-v15),Interaction(3,4);

Equal G, (v13), MainEffect(3):G, (v13), MainEffect(4):G, (v13), Interaction(3,4);
Coeff G, (v13), Interaction(3,4), -1;

Equal G, (v14), MainEffect(3):G, (v14), MainEffect(4):G, (v14), Interaction(3,4);
Coeff G, (v14), Interaction(3,4), -1;

Equal G, (v15), MainEffect(3):G, (v15), MainEffect(4):G, (v15), Interaction(3,4);
Coeff G, (v15), Interaction(3,4), -1;

```

Again, the maximum number of E-step and M-steps have been increased from the default, the E tolerance and M tolerance values have been decreased, and the SE calculation method has been set to REM in the <Options> section.

As noted in the output from the previous example, there were several latent dimensions that had no items loading onto them. Here, rather than using the general **InteractionEffects** keyword, we have used the more targeted **Generate** to construct only those interaction terms which will be used in the model. As seen in the Q-matrix and discussed earlier, only the interaction of attributes 3 and 4 is needed and is specified by the **Generate = (3,4);** statement. If additional interactions were needed, they may be added by placing a comma after the last interaction term and listing the other desired interactions. For example, if the 3-way interaction of attributes 2, 3 and 4 was desired in addition to the 3-4 interaction, we could specify that using a single generate statement of **Generate = (3,4), (2,3,4);**.

As before, we free the first 12 items onto the appropriate single attributes. Following the comment in the syntax, we free items 13 -15 on both main effect terms for attributes 3 and 4, as well as the interaction of the two attributes. To induce the equality specified in the model, we set the main effects and interaction terms equal to each of the items via a statement such as `Equal G, (v13), MainEffect(3):G, (v13), MainEffect(4):G, (v13), Interaction(3,4);` and then apply the -1 to the interaction with a coefficient constraint of the form `Coeff G, (v13), Interaction(3,4), -1;.`

**Output 6.5:** DINO Output - Item Parameters

Item	Label	P#	a	1	s.e.	P#	a	2	s.e.	P#	a	3	s.e.	P#	a	4	s.e.	P#	a	5	s.e.	P#	c	s.e.
1	v1	24	1.89	0.14	----		0.00	0.00	----		0.00	0.00	----		0.00	0.00	----	1	-1.68	0.08				
2	v2	25	1.90	0.15	----		0.00	0.00	----		0.00	0.00	----		0.00	0.00	----	2	-1.14	0.07				
3	v3	26	1.90	0.17	----		0.00	0.00	----		0.00	0.00	----		0.00	0.00	----	3	-0.63	0.06				
4	v4		0.00	----	27	2.47	0.26	0.17	0.26	----	0.00	0.00	----		0.00	0.00	----	4	-0.33	0.07				
5	v5		0.00	----	28	2.23	0.17	0.34	0.17	----	0.00	0.00	----		0.00	0.00	----	5	-1.15	0.08				
6	v6		0.00	----	29	1.91	0.34	0.00	0.00	----	0.00	0.00	----		0.00	0.00	----	6	1.55	0.07				
7	v7		0.00	----		0.00	----	30	1.84	0.13	0.00	0.00	----		0.00	0.00	----	7	-1.18	0.09				
8	v8		0.00	----		0.00	----	31	2.17	0.14	0.00	0.00	----		0.00	0.00	----	8	-0.79	0.09				
9	v9		0.00	----		0.00	----	32	1.68	0.13	0.00	0.00	----		0.00	0.00	----	9	-0.32	0.08				
10	v10		0.00	----		0.00	----		0.00	----	33	2.42	0.17		0.00	0.00	----	10	0.12	0.10				
11	v11		0.00	----		0.00	----		0.00	----	34	1.63	0.17		0.00	0.00	----	11	0.87	0.10				
12	v12		0.00	----		0.00	----		0.00	----	35	1.91	0.20		0.00	0.00	----	12	1.25	0.11				
13	v13		0.00	----		0.00	----		0.29	0.29	36	1.06	0.29	36	-1.06	0.29	0.29	13	2.74	0.21				
14	v14		0.00	----		0.00	----		0.20	0.20	37	1.51	0.20	37	-1.51	0.20	0.20	14	1.60	0.14				
15	v15		0.00	----		0.00	----		0.14	0.14	38	2.53	0.14	38	-2.53	0.14	0.14	15	-1.66	0.13				

As with the DINA model, the estimated item values reported are in the LCDM parameterization but DINO parameters of guessing and slippage may be obtained by conversion. Again, guessing is found from the reported parameters as  $\exp(c)/(1 + \exp(c))$  and one minus the slippage term  $(1 - s)$  is found as  $\exp(c + a)/(1 + \exp(c + a))$ . The  $a$  term used in the conversion should be the value reported in one of the main effect columns, not the negative value associated with the interaction term.

One aspect of the output that we haven't discussed as of yet is the model fit. We present the -2 log likelihood, AIC, and BIC values for each of the three models fit to our simulated data in Table 6.2. As with the output for more standard IRT models, these values are reported at the bottom of the -irt flexMIRT<sup>®</sup> output file. As can be seen in Table 6.2, the fit of the models indicates that using DINO for the final three items was the preferred model based on all three reported fit indices. It is informative to know that the same data set was used in all three examples and this data was generated using the DINO model for the last three items. The advantage for the DINO is sometimes small, but that is most likely a result of the majority of items being fit by single main effects, the specification of which did not vary regardless of the model that was used for the items with more complex attribute relationships.

**Table 6.2:** Model Fit for Basic CMDs Fit to Simulated Data

	CRUM	DINA	DINO
-2loglikelihood:	45799.53	45873.59	45798.31
Akaike Information Criterion (AIC):	45881.53	45949.59	45874.31
Bayesian Information Criterion (BIC):	46156.21	46204.17	46128.89

### Basic DINA with Testlet Structure

For the final example in this section, we employ a new simulated dataset that contains responses to 24 dichotomous items that are to be modeled with 4 attributes. In addition to being generated from a CDM, this data was simulated to incorporate a testlet structure, such as when multiple reading items use the same passage as a prompt. A graphical depiction of such a model was given in Figure 6.1, where  $y$ s are the observed items,  $x$ s are the discrete attributes, and  $\xi$ s are the testlet effects.

For the 24 item dataset we will be modeling, items 1 - 6 are members of testlet 1, items 7 - 12 belong to testlet 2, items 13-18 to testlet 3, and items

19-24 are members of the final testlet. The Q-matrix for mapping items onto attributes supplied in Table 6.3.

**Table 6.3:** Q-matrix for Testlet-DINA Example

Item	Attribute 1	Attribute 2	Attribute 3	Attribute 4
Item 1	1	1	0	0
Item 2	0	0	1	1
Item 3	1	0	1	0
Item 4	0	1	0	1
Item 5	1	0	0	1
Item 6	0	1	1	0
Item 7	1	1	0	0
Item 8	0	0	1	1
Item 9	1	0	1	0
Item 10	0	1	0	1
Item 11	1	0	0	1
Item 12	0	1	1	0
Item 13	1	1	0	0
Item 14	0	0	1	1
Item 15	1	0	1	0
Item 16	0	1	0	1
Item 17	1	0	0	1
Item 18	0	1	1	0
Item 19	1	1	0	0
Item 20	0	0	1	1
Item 21	1	0	1	0
Item 22	0	1	0	1
Item 23	1	0	0	1
Item 24	0	1	1	0

As can be seen in syntax, the `<Options>` section is largely unchanged from the previous examples, although we have reduced the number quadrature points from the default, due to the high-dimensional nature of this problem. In the first group, `G`, the data file, variable names, etc. have been given. As with previous CDM examples, we specify the number of attributes (`Attributes = 4;`) and also request that flexMIRT<sup>®</sup> create dimensions for the interaction effects. From the Q-matrix, every possible second order interaction is needed, so

we use `InteractionEffects = (2);` to generate those dimensions automatically. To incorporate the testlet structure into our model, we will be fitting a bifactor-type model, in which the CDM main effect and interactions are the primary/general dimensions and the testlet effects are considered the specific dimensions. To indicate this to flexMIRT<sup>®</sup> we set `Primary = 10;` which is the total number of CDM dimensions (4 main effects + 6 interactions) and set the total number of dimensions for the model equal to 14 (10 CDM dimensions + 4 testlet effect dimensions).

In the `<Constraints>` section, the first group of statements is used to assign items to the appropriate interaction terms, based on the Q-matrix, for the DINA model. The second group of constraints, beginning with `Free G, (v1-v6), Slope(11);` is used to assign items to their testlet dimension. As noted earlier, the first 10 latent dimensions are taken up by the CDM, so the first available dimension for a testlet effect is dimension 11, hence `Slope(11)` in the noted `Free` statement. The specific dimensions are specified as a restricted bifactor model, where the testlet effect is presumed to be a testlet-specific random intercept term. This requires the equality of the slopes within factors and is accomplished with the final group of constraints. With the final `Equal` constraint, we are fitting a Rasch-type model to the higher-order dimensions, constraining the slopes of attribute 1 through attribute 4 in group D to equality.

### Example 6.5: Testlet Structure with DINA model

```
<Project>
Title = "Fit HO-DINA Model with Testlets";
Description = "HO-DINA Model - 4 attributes";

<Options>
  Mode = Calibration;
  MaxE = 10000;
  Etol = 1e-4;
  Mtol = 1e-7;
  Quadrature = 25,6.0;
  Processors = 2;
  GOF = Extended;
  SE = REM;
  SaveDbg = Yes;
  SaveSco = Yes;
```

```

Score = EAP;

<Groups>

%G%
File = "fitDINAt.dat";
Varnames = v1-v24;
Ncats(v1-v24) = 2;
Model(v1-v24) = Graded(2);

Attributes = 4;
InteractionEffects = (2); //6 possible 2nd order ints.
Primary = 10; //number of dimensions to CDM
Dimensions = 14; //total dims, including testlet effects

%D%
Varnames = a1-a4;
DM = G;

<Constraints>
Fix G,(v1-v24),MainEffect;
Free G,(v1,v7,v13,v19),Interaction(1,2);
Free G,(v3,v9,v15,v21),Interaction(1,3);
Free G,(v5,v11,v17,v23),Interaction(1,4);
Free G,(v6,v12,v18,v24),Interaction(2,3);
Free G,(v4,v10,v16,v22),Interaction(2,4);
Free G,(v2,v8,v14,v20),Interaction(3,4);

Free G,(v1-v6),Slope(11);
Free G,(v7-v12),Slope(12);
Free G,(v13-v18),Slope(13);
Free G,(v19-v24),Slope(14);

Equal G,(v1-v6),Slope(11);
Equal G,(v7-v12),Slope(12);
Equal G,(v13-v18),Slope(13);
Equal G,(v19-v24),Slope(14);

Equal D,(a1-a4),Slope(1);

```

### 6.3.2 Replicating Published Examples

Because users are most likely acquainted with fitting CDMs in other programs, we also provide syntax and output for well-known examples to demonstrate that comparable parameter estimates may be obtained from flexMIRT® and benefits of either computation-time savings or simplicity of syntax are obtained.

The first example is a replication of Example 9.2 presented in *Diagnostic Measurement: Theory, Methods and Applications* by Rupp et al. (2010). The simulated data presented in this example uses 7 dichotomous items that measure 3 attributes. The Q-matrix given by the authors is duplicated here.

**Table 6.4:** Q-matrix for Rupp, Templin, & Henson's Example 9.2

Item	Attribute 1	Attribute 2	Attribute 3
Item 1	1	0	0
Item 2	0	1	0
Item 3	0	0	1
Item 4	1	1	0
Item 5	1	0	1
Item 6	0	1	1
Item 7	1	1	1

With the Q-matrix, we are now able to generate our flexMIRT® CDM syntax.

**Example 6.6:** Rupp, Templin, & Henson - Example 9.2

```
<Project>
Title = "Rupp Templin Henson Example 9.2";
Description = "Saturated LCDM model - 7 items, 3 attributes";

<Options>
  Mode = Calibration;
  MaxE = 20000;
  Etol = 1e-6;
  MaxM = 50;
  Mtol = 1e-9;
  GOF = Extended;
  SE = REM;
  SavePrm = Yes;
```

```

SaveCov = Yes;
SaveSco = Yes;
Score = EAP;

<Groups>

%G%
File = "ch9data.dat";
N = 10000;
Varnames = v1-v7,truec;
Select = v1-v7;
Ncats(v1-v7) = 2;
Model(v1-v7) = Graded(2);
Attributes = 3;
InteractionEffects = (2,3); // generate 2nd- and 3rd-order ints.
Dimensions = 7; // 3 main + 3 2nd-order + 1 3rd-order

%D%
DM = G;
Varnames = a;
Ncats(a) = 8;
Model(a) = Nominal(8);
Tc(a) = Identity;

<Constraints>
Fix G,(v1-v7),MainEffect;
Free G,(v1),MainEffect(1); // 1
Free G,(v2),MainEffect(2); // 2
Free G,(v3),MainEffect(3); // 3
Free G,(v4),MainEffect(1); // 1
Free G,(v4),MainEffect(2); // 2
Free G,(v4),Interaction(1,2); // 1x2
Free G,(v5),MainEffect(1); // 1
Free G,(v5),MainEffect(3); // 3
Free G,(v5),Interaction(1,3); // 1x3
Free G,(v6),MainEffect(2); // 2
Free G,(v6),MainEffect(3); // 3
Free G,(v6),Interaction(2,3); // 2x3
Free G,(v7),MainEffect(1); // 1
Free G,(v7),MainEffect(2); // 1

```

```

Free G,(v7),MainEffect(3); // 1
Free G,(v7),Interaction(1,2); // 1x2
Free G,(v7),Interaction(1,3); // 1x3
Free G,(v7),Interaction(2,3); // 2x3
Free G,(v7),Interaction(1,2,3); // 1x2x3

Fix D,(a),Slope;
Fix D,(a),ScoringFn;
Value D,(a),ScoringFn(1),0.0;

```

As with the previous CDM examples, the maximum number of E-step and M-steps have been increased from the default, the E tolerance and M tolerance values have been decreased, and the SE calculation method has been set to REM in the <Options> section.

The items were simulated as correct/incorrect items, so **Ncat=** has been set to 2 and we will fit the 2PLM to all items. As noted in the introduction, there are 3 attributes measured by the items, so **Attributes = 3**;. From the Q-matrix, we can see that Items 4-6 all measure two of the attributes and Item 7 measures all 3; to accommodate this aspect of the item assignment, we ask flexMIRT® to generate all second and third order effects for us via the **InteractionEffects** command. Finally, we set **Dimensions = 7**, as the total number of dimensions of the model is 3 main effects + 3 second order interactions + 1 third order interaction.

In the **D** group, we inform flexMIRT® that the diagnostic model we are setting up will be applied to group **G**, using the **DM = G**; statement. Within the **D** group, we specify that the attribute variable will be called **a** and that it has 8 categories, reflecting the 8 possible attribute profiles. Additionally, we tell flexMIRT® that the attribute variable will be fit with the nominal model and we use an identity matrix for the nominal model intercepts.

In the <Constraints> section, we assign items onto attributes according to the Q-matrix. First, we fix all items to 0 and then free items onto the appropriate main effect and interaction terms. For example, from the Q-matrix we see that Item 4 is informed by attributes 1 and 2 and the interaction of those attributes. In the <Constraints> section we free Item 4 with the statements **Free G,(v4), MainEffect(1); Free G,(v4), MainEffect(2);**, and **Free G,(v4), Interaction(1,2);**. The final three constraints are applied to the higher-level group, group **D**. The first two statements fix the slope and scoring function values and the final **Value** statement is used to fix the first scoring

function value to 0 for model identification purposes.

With the syntax created, we can now run the CDM in flexMIRT<sup>®</sup> and turn to the output.

## Output 6.6: Rupp, Templin, Henson Output - Item Parameters

```

Rupp Templin Henson Example 9.2
LCDM model - 7 items, 3 attributes

2PL Items for Group 1: G
Item      Label P#      a 1      s.e.      P#      a 2      s.e.      P#      a 3      s.e.      P#      a 4      s.e.      P#      a 5      s.e.      P#      a 6      s.e.      P#      a 7      s.e.      P#      c      s.e.
1          v1 15      1.87      0.06      0.00      0.00      0.00      0.00      0.00      0.00      0.00      0.00      0.00      0.00      0.00      0.00      0.00      0.00      0.00      0.00      0.00      0.00      0.00      0.00
2          v2          16      0.00      0.00      2.04      0.21      0.00      0.00      0.00      0.00      0.00      0.00      0.00      0.00      0.00      0.00      0.00      0.00      0.00      0.00      0.00      0.00      0.00
3          v3          17      0.00      0.00      0.00      0.00      0.00      0.00      0.00      0.00      0.00      0.00      0.00      0.00      0.00      0.00      0.00      0.00      0.00      0.00      0.00      0.00      0.00
4          v4 18      2.05      0.17      1.86      0.20      0.00      0.00      2.01      0.14      0.00      1.03      0.34      0.00      0.00      0.00      0.00      0.00      0.00      0.00      0.00      0.00      0.00
5          v5 21      1.79      0.33      0.00      0.00      0.00      0.00      1.72      0.36      0.00      20      1.38      0.58      0.00      0.00      0.00      0.00      0.00      0.00      0.00      0.00      0.00
6          v6          22      0.00      0.00      2.15      0.12      2.10      0.11      0.00      0.00      0.00      23      1.38      0.58      0.00      0.00      0.00      0.00      0.00      0.00      0.00      0.00      0.00
7          v7 27      2.47      0.22      2.13      0.26      2.06      0.28      0.00      0.00      0.00      30      0.81      0.42      0.00      0.00      0.00      0.00      0.00      0.00      0.00      0.00      0.00
7          v7 27      2.47      0.22      2.13      0.26      2.06      0.28      0.00      0.00      0.00      31      0.81      0.42      0.00      0.00      0.00      0.00      0.00      0.00      0.00      0.00      0.00
7          v7 27      2.47      0.22      2.13      0.26      2.06      0.28      0.00      0.00      0.00      32      0.81      0.42      0.00      0.00      0.00      0.00      0.00      0.00      0.00      0.00      0.00
7          v7 27      2.47      0.22      2.13      0.26      2.06      0.28      0.00      0.00      0.00      33      0.81      0.42      0.00      0.00      0.00      0.00      0.00      0.00      0.00      0.00      0.00

Diagnostic IRT Attributes and Cross-classification Probabilities for Group 1: G
Pattern      Prob
0 0 0      0.24810019
0 0 1      0.07355267
0 1 0      0.09927395
0 1 1      0.09119598
1 0 0      0.08634180
1 0 1      0.09134911
1 1 0      0.09358603
1 1 1      0.21660027

Rupp Templin Henson Example 9.2
LCDM model - 7 items, 3 attributes

GPC Items for Group 2: D
Item      Label P#      a      s.e.      b      s.e.      d 1      s.e.      d 2      s.e.      d 3      s.e.      d 4      s.e.      d 5      s.e.      d 6      s.e.      d 7      s.e.      d 8      s.e.
1          a          0.00      0.00      0.00      0.00      0.00      0.00      0.00      0.00      0.00      0.00      0.00      0.00      0.00      0.00      0.00      0.00      0.00      0.00      0.00      0.00      0.00

Nominal Model Slopes and Scoring Function Contrasts for Items for Group 2: D
Item      Label P#      a      s.e.      P# alpha 1      s.e.      P# alpha 2      s.e.      P# alpha 3      s.e.      P# alpha 4      s.e.      P# alpha 5      s.e.      P# alpha 6      s.e.      P# alpha 7      s.e.
1          a          0.00      0.00      Trend      1.00      0.00      0.00      0.00      0.00      0.00      0.00      0.00      0.00      0.00      0.00      0.00      0.00      0.00      0.00

Nominal Model Scoring Function Values Group 2: D
Item      Label      s 1      s 2      s 3      s 4      s 5      s 6      s 7      s 8
1          a          0.00      1.00      2.00      3.00      4.00      5.00      6.00      7.00

Nominal Model Intercept Contrasts for Items for Group 2: D
Item      Label Contrasts P# gamma 1      s.e.      P# gamma 2      s.e.      P# gamma 3      s.e.      P# gamma 4      s.e.      P# gamma 5      s.e.      P# gamma 6      s.e.      P# gamma 7      s.e.
1          a Identity      8      -1.22      0.52      9      -0.92      0.43      10      -1.00      0.13      11      -1.06      0.31      12      -1.00      0.37      13      -0.97      0.24      14      -0.14      0.05

Original (Bock, 1972) Parameters, Nominal Items for Group 2: D
Item      Label Category:      1      2      3      4      5      6      7      8
1          a          0.00      0.00      0.00      0.00      0.00      0.00      0.00      0.00
1          c          -0.00      -1.22      -0.92      -1.00      -1.06      -1.00      -0.97      -0.14

```

The output section labeled **2PL Items for Group 1: G** presents the item parameters for the 7 items, with estimated versus fixed at zero loadings mirroring the Q-matrix specifications, with the factors labeled a1-a3 representing main effects for attributes 1 - 3 and a4 - a7 representing the 2nd and 3rd order interactions. For those wishing to compare flexMIRT<sup>®</sup> to MPLUS (the program used to estimate the book example), this item parameter section corresponds with the “New/Additional Parameters” section of MPLUS output and the point estimates agree with those given in Figure 9.15 (pg. 218) of Rupp et al. (2010).

The next section of interest is labeled **Diagnostic IRT Attributes and Cross-classification Probabilities for Group 1: G**, which reports the estimated proportions of respondents in each attribute profile. This section in flexMIRT<sup>®</sup> corresponds to the MPLUS output section labeled **FINAL CLASS COUNTS AND PROPORTIONS FOR THE LATENT CLASSES...** and the reported values match those given in Figure 9.13 of Rupp et al. (2010).

There are some difference between flexMIRT<sup>®</sup> and MPLUS. For example, MPLUS fixes the latent variable mean of the last latent class (attribute profile 8 in this case) to 0, while in flexMIRT<sup>®</sup> we have constrained the first latent class to 0. This results in differences in the latent attribute parameter estimates, but as we show in the below table these differences are simply a matter of choice of identification constraints and do not affect the primary estimates of interests (i.e., estimated profile proportions).

**Table 6.5:** Generating latent class mean values and flexMIRT<sup>®</sup> and MPLUS estimates for Rupp, Templin, & Henson’s Example 9.2

	c1	c2	c3	c4	c5	c6	c7	c8	sum
generating									
$\mu$	0	-1.00	-1.00	-1.00	-1.00	-1.00	-1.00	0	
$\exp(\mu)$	1	0.367879	0.367879	0.367879	0.367879	0.367879	0.367879	1	4.207277
$v$	0.2481	0.0913	0.0913	0.0913	0.093	0.0913	0.0913	0.2481	
flexMIRT									
$\hat{\mu}$	0	-1.21583	-0.91595	-1.00082	-1.05552	-0.99914	-0.97495	-0.13578	
$\exp(\hat{\mu})$	1	0.296464	0.400136	0.367578	0.348011	0.368196	0.377211	0.87303468	4.030631
$\hat{v}$	0.2481	0.0736	0.0993	0.0912	0.0863	0.0914	0.0936	0.2166	
MPLUS									
$\hat{\mu}$	0.136	-1.08	-0.78	-0.865	-0.92	-0.863	-0.839	0	
$\exp(\hat{\mu})$	1.145682	0.339596	0.458406	0.421052	0.398519	0.421894	0.432142	1	4.617291
$\hat{v}$	0.2481	0.0736	0.0993	0.0912	0.0863	0.0914	0.0936	0.2166	

We will next cover the contents of the requested file that contains the individual scores.

**Output 6.7:** Rupp, Templin, Henson Output - -sco File

1	1	0.162486	0.019853	0.598993	0.368896	0.139494	0.490102	0.136084	-0.002661	0.019459	-0.018766	-0.008690	0.240200	\\
0.300930	0.517296	0.016153	0.003135	0.083426	0.078495	0.000499	0.000066	2	0.000000	1.000000	1.000000	-0.001504	0.034935	\\
1	0.041052	0.047476	0.036249	0.198411	0.212654	0.186910	0.039367	-0.001706	0.045222	-0.001260	-0.000000	-0.001504	0.034935	\\
0.875910	0.035804	0.047016	0.000217	0.040582	0.000228	0.000243	0.000000	1	0.000000	1.000000	1.000000	-0.024751	0.134896	\\
1	0.198872	0.190374	0.160731	0.399151	0.392596	0.367282	0.159322	-0.019122	0.154132	-0.024583	-0.000000	-0.001504	0.034935	\\
0.481882	0.147611	0.165898	0.005738	0.172862	0.007272	0.018627	0.000110	1	0.000000	1.000000	1.000000	-0.007338	0.089377	\\
1	0.041052	0.047476	0.036249	0.198411	0.212654	0.186910	0.039367	-0.001706	0.045222	-0.001260	-0.000000	-0.001504	0.034935	\\
0.875910	0.035804	0.047016	0.000217	0.040582	0.000228	0.000243	0.000000	1	0.000000	1.000000	1.000000	-0.007338	0.089377	\\
1	0.475195	0.091060	0.900778	0.499384	0.287695	0.288960	0.249385	-0.031539	0.082768	-0.008865	-0.000000	-0.000300	0.021774	\\
0.029570	0.415907	0.013637	0.065691	0.063279	0.410184	0.002736	0.008996	2	0.000000	1.000000	1.000000	-0.000300	0.021774	\\
1	0.880070	0.979322	0.977730	0.324880	0.142303	0.147561	0.105547	-0.001801	0.020250	-0.001495	-0.000000	-0.002010	0.130137	\\
0.000006	0.000673	0.001170	0.118081	0.000154	0.019844	0.020940	0.839131	8	0.000000	1.000000	1.000000	-0.002010	0.130137	\\
1	0.981067	0.979636	0.846212	0.136289	0.141243	0.360745	0.018575	-0.000279	0.019950	-0.001603	-0.000000	-0.005747	0.012509	\\
0.000007	0.000100	0.001301	0.017525	0.001116	0.019142	0.151364	0.809445	8	0.000000	1.000000	1.000000	-0.005747	0.012509	\\
1	0.150848	0.679496	0.012669	0.357900	0.466670	0.111843	0.128093	-0.034142	0.217781	-0.001397	-0.000000	-0.028909	0.230886	\\
0.228668	0.009347	0.608329	0.002808	0.082029	0.000460	0.068305	0.000054	3	0.000000	1.000000	1.000000	-0.028909	0.230886	\\
1	0.751563	0.081414	0.638255	0.432107	0.273469	0.480505	0.186716	-0.029184	0.074785	0.013218	-0.000000	0.000838	0.081112	\\
0.071059	0.127968	0.032030	0.017380	0.232326	0.487234	0.026330	0.005674	6	0.000000	1.000000	1.000000	0.000838	0.081112	\\
1	0.143402	0.935046	0.910959	0.350483	0.246444	0.284802	0.122838	-0.025556	0.060735	-0.031276	-0.000000	0.000838	0.081112	\\
0.001455	0.028628	0.043541	0.782973	0.005167	0.029704	0.038878	0.069654	4	-0.000000	1.000000	1.000000	0.000838	0.081112	\\

As in the previous -sco output, we have given the first 10 cases and have added a line break and separators. The first two columns of the -sco file give the group and observation number, similar to a standard flexMIRT<sup>®</sup> IRT score output file. Recall that for this example, there are 3 attributes and 8 possible attribute profiles. The probability of the respondent having mastered each of the attributes individually is given, starting after the observation number. For example, for observation 1, the probability they have mastery of attribute 1 is 0.16, for attribute 2 it is 0.02 and for attribute 3 it is 0.60. The next three columns are the estimated SEs for those probabilities, respectively. The last six entries on the first line of each observation contain the entries for the estimated attribute variance/covariance matrix.

Finally, on what has been moved to the second line for each respondent (but is a continuation of the same in the actual -sco file), the posterior probabilities for each of the attribute profiles (8 possible for this example) are given. For observation 1, the posterior probability of being in attribute profile 1 is 0.30, the probability of being in attribute profile 2 is 0.52 and so on. The order in which the probabilities are given matches the order used in the output table “Diagnostic IRT Attributes and Cross-classification Probabilities.” Referring to the output presented for this example, we find that attribute profile 1 corresponds to 0 0 0 (meaning no attributes are mastered) and attribute profile 2 corresponds to 0 0 1 (meaning only attribute 3 has been mastered), etc.. After the probabilities for each of the attribute profiles are presented, the most likely attribute profile for the respondent is given; for respondent 1 that is attribute profile 2, for respondents 6 and 7, that is attribute profile 8. The last three values given in the -sco file are the higher-order latent variable point estimate followed by the SD and variance for that estimate. In this case, all individual theta estimates for the higher-order variable are zero and all SD and variance estimates are 1.00 because the nominal model was fit, which has zero slope and is a reparameterization of the profile probabilities.

### **Unstructured Higher-Order Latent Space**

It is also possible to fit CDMs in flexMIRT<sup>®</sup> without using the higher-order latent variable at all. This is accomplished by removing all references to the D group in setting up the model. We present this syntax below. The only changes are the removal of the %D% group from the <Groups> section as well as omitting the parameter constraints that were applied to that group.

**Example 6.7:** Rupp, Templin, & Henson - Example 9.2 without D group

```
<Project>
Title = "Rupp Templin Henson Example 9.2 redone";
Description = "LCDM model - 7 items, 3 attributes:";

<Options>
  Mode = Calibration;
  MaxE = 20000;
  Etol = 1e-6;
  MaxM = 50;
  Mtol = 1e-9;
  GOF = Extended;
  SE = REM;
  SavePrm = Yes;
  SaveCov = Yes;
  SaveSco = Yes;
  Score = EAP;

<Groups>

%G%
  File = "ch9data.dat";
  N = 10000;
  Varnames = v1-v7,truec;
  Select = v1-v7;
  Ncats(v1-v7) = 2;
  Model(v1-v7) = Graded(2);
  Attributes = 3;
  InteractionEffects = (2,3); // generate 2nd- and 3rd-order ints.
  Dimensions = 7; // 3 main + 3 2nd-order + 1 3rd-order

<Constraints>
  Fix G,(v1-v7),MainEffect;
  Free G,(v1),MainEffect(1); // 1
  Free G,(v2),MainEffect(2); // 2
  Free G,(v3),MainEffect(3); // 3
  Free G,(v4),MainEffect(1); // 1
  Free G,(v4),MainEffect(2); // 2
  Free G,(v4),Interaction(1,2); // 1x2
```

```

Free G,(v5),MainEffect(1); // 1
Free G,(v5),MainEffect(3); // 3
Free G,(v5),Interaction(1,3); // 1x3
Free G,(v6),MainEffect(2); // 2
Free G,(v6),MainEffect(3); // 3
Free G,(v6),Interaction(2,3); // 2x3
Free G,(v7),MainEffect(1); // 1
Free G,(v7),MainEffect(2); // 1
Free G,(v7),MainEffect(3); // 1
Free G,(v7),Interaction(1,2); // 1x2
Free G,(v7),Interaction(1,3); // 1x3
Free G,(v7),Interaction(2,3); // 2x3
Free G,(v7),Interaction(1,2,3); // 1x2x3

```

With respect to the output, presented next, while there are some slight changes in the estimates, the reported point estimate values for the item parameters remain largely unchanged from the calibration that employed the higher-order latent variable. Differences in the estimated profile probabilities are not seen until the 4th or 5th decimal place. If one looked at the -sco file, there are also only slight changes to the reported values; the most noticeable difference between the -sco file from the model employing the higher-order variable and the current example is that final three values discussed in the previous -sco file (the higher-order latent variable mean, SD, and variance estimates) are no longer present.

**Output 6.8:** Rupp, Templin, Henson Output - Item Parameters without D group

2PL Items for Group 1: G																																	
Item	Label	P#	a	1	s.e.	P#	a	2	s.e.	P#	a	3	s.e.	P#	a	4	s.e.	P#	a	5	s.e.	P#	a	6	s.e.	P#	a	7	s.e.	P#	c	s.e.	
1	v1	8	1.87	0.07	---	---	0.00	---	---	---	0.00	---	---	---	0.00	---	---	---	---	0.00	---	---	0.00	---	---	---	---	---	---	---	1	-0.91	0.04
2	v2	---	0.00	---	---	9	2.04	0.07	---	---	0.00	---	---	---	0.00	---	---	---	---	0.00	---	---	0.00	---	---	---	---	---	---	2	-1.01	0.04	
3	v3	---	0.00	---	---	---	0.00	---	---	10	2.01	0.00	---	---	0.00	---	---	---	---	0.00	---	---	0.00	---	---	---	---	---	---	3	-0.96	0.04	
4	v4	11	2.05	0.18	12	1.86	0.18	---	---	---	0.00	---	---	13	1.03	0.31	0.00	---	---	0.00	---	---	0.00	---	---	---	---	---	---	4	-2.44	0.13	
5	v5	14	1.79	0.17	---	---	0.00	---	---	---	0.00	---	---	15	1.72	0.17	0.00	---	---	16	1.38	0.32	0.00	---	---	---	---	---	---	5	-2.24	0.11	
6	v6	---	0.00	---	---	---	0.00	---	---	17	2.15	0.19	18	2.10	0.20	0.00	---	---	---	0.00	---	---	---	---	---	---	---	---	---	6	-2.54	0.14	
7	v7	20	2.47	0.57	21	2.13	0.58	22	2.06	0.64	23	0.81	0.80	0.80	0.75	0.85	25	0.98	1.11	0.37	0.00	---	---	---	---	---	---	---	7	-3.63	0.43		
Diagnostic IRT Attributes and Cross-classification Probabilities for Group 1: G																																	
Pattern Prob																																	
0	0	0	0.24809775																														
0	1	0	0.07355411																														
0	1	0	0.09927471																														
0	1	1	0.09119567																														
1	0	0	0.08634185																														
1	0	1	0.09134873																														
1	1	0	0.09358578																														
1	1	1	0.21660141																														

## de la Torre Subtraction Example

In a demonstration of a higher order latent trait CDM to non-simulated data, de la Torre and Douglas (2004) fit a higher-order DINA model to a 20-item fraction subtraction test given to 2144 examinees. In the noted paper, an 8 attribute model was estimated using MCMC, with final parameter estimates based on averaging the estimates from 10 parallel chains, each with 20000 iterations of which the first 10000 were discarded as burn-in. We provide syntax and output of a flexMIRT<sup>®</sup> replication of this analysis.

### Example 6.8: de la Torre Subtraction Syntax

```
<Project>
Title = "Tatsuoka Subtraction Data";
Description = "Restricted Higher-order DINA Model";

<Options>
  Mode = Calibration;
  MaxE = 20000;
  MaxM = 5;
  Mtol = 0;
  Etol = 1e-5;
  GOF = Extended;
  SE = REM;
  SavePrm = Yes;
  SaveCov = Yes;
  SaveSco = Yes;
  Score = EAP;
  Processors = 4;
  NewThreadModel = Yes;

<Groups>

%G%
  File = "subtraction.csv";
  Varnames = v1-v20;
  Ncats(v1-v20) = 2;
  Model(v1-v20) = Graded(2);
  Attributes = 8;
  Generate = (3,6,7),(4,7),(2,3,5,7),(2,4,7,8),(1,2,7),(2,5,7,8),
```

```

(2,5,7),(7,8),(2,4,5,7), (2,7),(1,7),
(2,5,6,7),(1,2,3,5,7),(2,3,5,7);
  Dimensions = 22; // 8 main effects + 14 generated higher ord ints

%D%
  DM = G;
  Varnames = a1-a8;

<Constraints>
  Fix G,(v1-v20),MainEffect;
  Free G,(v1),Interaction(3,6,7); //3rd-order int of attr 3,6,7
  Free G,(v2),Interaction(4,7); //2nd-order int of attr 4,7
  Free G,(v3),Interaction(4,7); //2nd-order int of attr 4,7
  Free G,(v4),Interaction(2,3,5,7); //4th-order int of attr 2,3,5,7
  Free G,(v5),Interaction(2,4,7,8); //4th-order int of attr 2,4,7,8
  Free G,(v6), MainEffect(7); //main effect of attr 7
  Free G,(v7),Interaction(1,2,7); //3rd-order int of attr 1,2,7
  Free G,(v8), MainEffect(7); // main effect of attr 7
  Free G,(v9), MainEffect(2); // main effect of attr 2
  Free G,(v10),Interaction(2,5,7,8); //4th-ord int of attr 2,5,7,8
  Free G,(v11),Interaction(2,5,7); //3rd-order int of attr 2,5,7
  Free G,(v12),Interaction(7,8); // 2nd-order int of attr 7,8
  Free G,(v13),Interaction(2,4,5,7); //4th-ord int of attr 2,4,5,7
  Free G,(v14),Interaction(2,7); //2nd-order int of attr 2,7
  Free G,(v15),Interaction(1,7); //2nd-order int of attr 1,7
  Free G,(v16),Interaction(2,7); //2nd-order int of attr 2,7
  Free G,(v17),Interaction(2,5,7); //3rd-order int of attr 2,5,7
  Free G,(v18),Interaction(2,5,6,7); //4th-ord int of attr 2,5,6,7
  Free G,(v19),Interaction(1,2,3,5,7); //5th-ord int: att 1,2,3,5,7
  Free G,(v20),Interaction(2,3,5,7); //4th-order int: att 2,3,5,7

  Equal D,(a1-a8),Slope; // "restricted" higher order model

```

As with the previous CDM example, the maximum number of E-steps has been increased from the default, the E tolerance and M tolerance values have been decreased, and the SE calculation method has been set to REM in the <Options> section. As noted earlier, these changes are made primarily to improve the resulting SE estimates. We have also requested the newly implemented multi-core processing model be used for this analysis via `NewThreadModel = Yes;`. This model is requested due to the high-

dimensional nature of the current problem. Additional information regarding the new thread model is available in the Syntax chapter.

In the first group section, have used the more targeted **Generate** to construct only those interaction terms which will be used in the model. The needed interaction effects are generated in the order they will be used for the items (e.g., item 1 requires the interaction of attributes 3, 6, and 7) but this not necessary - as noted earlier, as many interactions as needed may be constructed from a single **Generate** statement and the order in which interactions are listed does not matter. Counting the interactions specified in the **Generate** statement, we find there are 14 interactions needed to correctly assign items to correspond to the Q-matrix. Adding the 14 interactions with the 8 main effects gives us a total of 22 latent dimensions in the model.

In the **<Constraints>** section, all main effect terms are initially fixed to 0 and then **Free** statements are added to replicate the Q-matrix for the problem (which may be found in Table 8 on pg. 347 of de la Torre & Douglas, 2004). Because a DINA is fit to all items, only the highest-order interaction of attributes for each item is specified and main effect terms are specified for only those items that depend on a single attribute (i.e., items 6, 8, and 9). The last statement in the **<Constraints>** section sets the latent attribute slope parameters to be equal across attributes, producing a restricted higher-order model.

As with all examples, the full output file is available on the program's Support site. The item parameter table for this example is extremely wide (22 latent dimension columns, an intercept column, and all associated SEs) and is not presented here due to size. However, we have summarized the flexMIRT<sup>®</sup> parameter estimates in Table 6.6. Please note that the slopes are presented in a single column in this table for efficiency of space; the estimated slope values are not all on the same latent dimension, as can be seen in the full output file. In addition to the flexMIRT<sup>®</sup> estimates, we also present the original slippage and guessing parameter values reported by de la Torre and Douglas (2004) in their Table 9 and the flexMIRT<sup>®</sup> estimates converted to this parameterization. Comparing the point estimates across the two programs, one notices some differences but overall the values are quite similar. Regressing the original guessing parameter estimates onto the converted flexMIRT<sup>®</sup> values gives an intercept value close to zero (0.006), a regression coefficient on 0.97, and an  $R^2$  value of 0.99. A similar comparison of the slippage values also results in a low intercept (0.01), a regression coefficient near 1 (0.99), and a high  $R^2$  value (0.98). Given the comparability of the parameter

values, it is interesting to note that flexMIRT<sup>®</sup> completed its calibration in 20.5 seconds, highlighting the extreme efficiency that may be obtained when estimating higher-order CDMs via MML.

**Table 6.6:** Guessing and Slippage Parameters from flexMIRT and MCMC

Item	flexMIRT				de la Torre & Douglas	
	Intercept (c)	Slope (a)	g	1 - s	g	1 - s
v1	-4.59	6.50	0.01	0.87	0.04	0.90
v2	-3.42	6.90	0.03	0.97	0.03	0.96
v3	-4.49	6.47	0.01	0.88	0.00	0.88
v4	-1.25	3.21	0.22	0.88	0.22	0.89
v5	-0.84	2.35	0.30	0.82	0.30	0.82
v6	-4.99	8.09	0.01	0.96	0.03	0.96
v7	-3.86	5.25	0.02	0.80	0.03	0.81
v8	-0.20	1.65	0.45	0.81	0.44	0.81
v9	-1.63	2.75	0.16	0.75	0.18	0.75
v10	-3.51	4.82	0.03	0.79	0.03	0.79
v11	-2.66	5.24	0.07	0.93	0.07	0.93
v12	-1.91	4.74	0.13	0.94	0.13	0.96
v13	-4.30	5.02	0.01	0.67	0.02	0.67
v14	-3.20	5.79	0.04	0.93	0.05	0.93
v15	-3.43	5.51	0.03	0.89	0.04	0.90
v16	-2.25	4.23	0.10	0.88	0.10	0.88
v17	-3.00	4.83	0.05	0.86	0.04	0.86
v18	-1.81	3.66	0.14	0.86	0.13	0.85
v19	-3.82	4.87	0.02	0.74	0.02	0.76
v20	-4.14	5.68	0.02	0.82	0.01	0.84

Note: g is found from flexMIRT parameters as  $\exp(c)/(1 + \exp(c))$  and  $1 - s$  is found as  $\exp(c + a)/(1 + \exp(c + a))$ .

## CHAPTER 7

---

### Details of the Syntax

---

This section provides detailed coverage of all the syntax statements and associated options that are available in flexMIRT<sup>®</sup>. Required statements are listed in bold, optional statements are in normal font. When keywords are required with statements, the possible options are listed after the statement and the default setting is underlined. If numeric values are required with a statement, such as for a convergence criterion, the default value is presented. If there is no default value, a question mark will denote where values need be entered.

---

#### 7.1. The Project Section

**Example 7.1:** <Project> section commands

<pre><b>Title</b> = " "; <b>Description</b> = " ";</pre>
--

Both of these statements must be present in the <Project> section, although the actual content within the quotes may be left blank.

---

#### 7.2. The Options Section

The <Options> section is where the type of analysis to be conducted is specified and where technical details of the analysis may be modified, including convergence criteria, scoring methods, and the level of detail desired in the output. As indicated by the bold, the **Mode** statement is the only required command and determines the type of analysis that will be conducted. If **Mode = Scoring**; or **Mode = Simulation**; is selected, some additional technical commands, covered in the next group of statements become required. The setting of **Progress** determines if flexMIRT<sup>®</sup> will print detailed progress in-

formation in the console window, (e.g., iteration number and log-likelihood value). **TechOut** determines if controls values such as tolerance values, processing times, names of outputted files, etc. will be printed in the preamble of the output. **NumDec** determines the number of decimal places reported for item parameters in the output file - the default is 2. The other commands allow for the optional saving of additional output into separate files. **SaveCov** refers to the covariance matrix of the parameter estimates; **SaveInf**, the Fisher information function values of the items and the test; **SavePrm**, the item and group parameter estimates; **SaveSco**, the individual IRT scale scores, and **SaveDbg**, additional technical information. Each of these additional output requests are saved to files with corresponding extensions; that is, the **SaveInf** command results in an output file with “\*-inf.txt” appended to the command file name; **SaveCov** appends “\*-cov.txt”, etc.

**Example 7.2:** <Options> - Engine Mode and Output Statements

```
<Options >
Mode = Calibration/Scoring/Simulation;
Progress = Yes/No;
TechOut = Yes/No;
NumDec = 2;
SaveCov = Yes/No;
SaveInf = Yes/No;
SavePrm = Yes/No;
SaveSco = Yes/No;
SaveDbg = Yes/No;
DMtable = Yes/No;
NormalMetric3PL = Yes/No;
SlopeThreshold = Yes/No;
```

Specific to CDMs, the command **DMtable** controls whether the diagnostic classification probability table is printed in the \*-irt file. The default is to print the table, but this may be modified by changing the command to **DMtable=No**;. Modifying this setting may be helpful if the probability table will be large.

The final two statements in this section are provided primarily to facilitate the use of parameter estimates obtained from other programs, but users may find them convenient for other purposes. Both **NormalMetric3PL** and **SlopeThreshold** are used to print, save out, or read-in parameters that are

in more traditional IRT metrics than the MIRT-compatible logistic metric flexMIRT<sup>®</sup> uses; these keywords are only available for use in conjunction with unidimensional models.

When **NormalMetric3PL = Yes;**, the program will print normal-metric parameter estimates in the output and save those values into the -prm file, if requested. Specifically, flexMIRT<sup>®</sup> will 1) print/save ( $a/1.702$ ) rather than the typical logistic metric slope values, 2) instead of the intercept ( $b * -a$ ) flexMIRT<sup>®</sup> will print/output the difficulty (b) parameter value, and 3) rather than the logit-guessing parameter, the program will report/save the customary  $g$  value. Note that even if the normal metric keyword is used during calibration, any priors that may be specified are applied to the MIRT-compatible metrics (prior applied to intercept, not difficulty; normal guessing prior still applied to logit- $g$  distribution, etc). If **NormalMetric3PL** is set to **Yes** for a scoring run, then flexMIRT<sup>®</sup> will expect the all parameter values read-in from the -prm file for scoring to be in the normal metric.

By default, flexMIRT<sup>®</sup> prints both the intercept and difficulty parameters in the output for unidimensional models, but saves only the intercept values into a requested -prm file. When **SlopeThreshold = Yes;**, rather than saving the intercept values, the program will write the difficulty (b) values to the -prm file. If **SlopeThreshold** is set to **Yes** in conjunction with a 3PL model, the psuedo-guessing parameters ( $gs$ ) will be printed/saved, rather than the default logit-guessing parameter values. Users should be aware that, unlike the behavior when **NormalMetric3PL** is used, the **SlopeThreshold** keyword has no effect on the metric of the slope;  $a$  values printed to the -prm are in the logistic metric, regardless of the **SlopeThreshold** setting.

### Example 7.3: <Options> - Technical Specifications

```
<Options >
...
Score = EAP/MAP/ML/SSC;
MaxMLscore = ?;
MinMLscore = ?;
Mstarts = Yes/No;
Rndseed =?;
ReadPRMFile = "*.txt";
SE = Xpd/Mstep/SEM/Sandwich/Fisher/FDM/REM;
SmartSEM = Yes/No;
Perturbation = 0.001;
FisherInf = 1, 0.0;
PriorInf = Yes/No;
logDetInf = Yes/No;
Quadrature = 49, 6.0;
MaxE = 500;
MaxM =100;
Etol = 1e-4;
Mtol = 1e-9;
SEMtol =1e-3;
SStol =1e-4;
Processors = 1;
NewThreadModel = Yes/No;
```

As detailed previously, when **Mode** is set to something other than **Calibration**, additional statements are required. It was noted earlier that specifying **SaveSC0** = **Yes** during a calibration run triggers a combined scoring run. If a scoring run is specified, either by explicitly setting **Mode** to **Scoring** or implicitly by requesting a combined scoring run, an IRT scoring method is required. As may be deduced from the lack of an underlined scoring option, there is no default. The options available for scoring are Expected *A Posteriori* (keyword is **EAP**), Maximum *A Posteriori* (keyword is **MAP**), Maximum Likelihood (keyword is **ML**), and Summed Score Conversions to EAP scores (keyword is **SSC**), which in addition to individual scores provides a conversion table in the output that translates summed scores to EAP scale scores.

When using ML scoring, there are two additional keywords that must be employed. Users are required specify the desired maximum and minimum

scores to be assigned by flexMIRT<sup>®</sup> using the **MaxMLscore** and **MinMLscore** keywords, respectively. There are no default values for the ML minimum and maximum scores. **Efforts have been made to make ML scoring as robust as possible, but it is not recommended for use with multi-dimensional models. Because ML scoring does not use information from the population distribution, essential statistical information about the population distribution (e.g., factor inter-correlations, means, and variances) is ignored when scoring individual dimensions of MIRT models. Additionally, ML scoring can lead to score information matrices that are not positive definite, making the SEs for some estimates undefined.**

flexMIRT<sup>®</sup> offers a multiple dispersed starting value option for use with ML and MAP scoring runs. For unidimensional models, the default theta starting value in MAP/ML scoring is 0. When **Mstarts = Yes**; flexMIRT<sup>®</sup> will do 12 additional restarts using a range of starting values from -2.75 to 2.75, in step sizes of 0.5. The best solution (as determined by the highest log-likelihood) will be picked from the 13 values. Note that using **Mstarts = Yes**; will increase the time needed to complete scoring, but provides protection against local modes in 3PL or nominal model scoring.

When a **Simulation** run is specified by the **Mode** command, the random number generator must be “seeded”. An integer value must be explicitly provided using **Rndseed** command. There is no default value.

The **ReadPRMFile** command is used to read parameters values from a text file to flexMIRT<sup>®</sup>, which may be used as starting values for a calibration run, as fixed item parameters for a scoring run, and as true generating parameter values for a simulation run. The proper layout for a parameter file is covered in detail in Chapter 4: Simulation.

flexMIRT<sup>®</sup> has several options available for computing the standard errors of estimated item/group parameters. The standard error method may be changed by the user through the **SE** command. The default method is empirical cross-product approximation (keyword is **Xpd**). Standard errors may also be calculated via the supplemented EM algorithm (keyword is **SEM**; Cai, 2008), from the Fisher (expected) information matrix (keyword is **Fisher**), via the sandwich covariance matrix (keyword is **Sandwich**), using the forward difference method (keyword is **FDM**; e.g., Jamshidian & Jennrich, 2000), or from the Richardson extrapolation method (keyword is **REM**; e.g., Jamshidian & Jennrich, 2000). **Mstep** values from the EM algorithm can be requested although, technically, the M-step standard errors are incorrect. They tend to be too

small, but under some circumstances (e.g., huge calibration sample size and thousands of item parameters), it may be more efficient to bypass the more elaborate (and correct) standard error computations and focus only on item parameter point estimates.

As an additional note, three of the available standard error methods have additional options that may be adjusted by the user. By default the supplemented EM algorithm SEs come from an optimized window of iterations. If you find that the SEs are failing to converge in that optimized window, the optional command **SmartSEM** may be useful. By setting **SmartSEM = No**; it will allow flexMIRT<sup>®</sup> to use the full EM iteration history, which may lead to a better window and converged standard errors.

The **Perturbation** command controls a value used by both the forward difference and Richardson extrapolation methods of SE estimation. These methods require the selection of a perturbation constant that is used in the denominator of the finite difference approximation; the choice of this perturbation constant may exert substantial influence on the accuracy of the final item parameter SE estimates (Tian, Cai, Thissen, & Xin, 2012). As noted above, the flexMIRT<sup>®</sup> default of this perturbation constant is set to 0.001, the value found to produce the consistently least poor estimates from the two methods when studied in both simulations and as applied to real data (Tian et al., 2012). While the forward difference and Richardson extrapolation methods are available to users, Tian et al. (2012) found them to be less accurate than the supplemented EM algorithm SEs when applied to standard IRT models, with no noticeable savings with respect to time when the smart window for the supplemented EM SEs was employed.

The information function for the individual items and full scale is printed in the general output (i.e., the “\*-irt.txt” file) using theta values from -2.8 to 2.8, changing by steps of 0.4. The **SaveInf** option addressed earlier allows the user to output the information function values to a separate “\*-inf.txt” file, which is useful for additional computations external to flexMIRT<sup>®</sup> or when information functions are plotted. By default, the “\*-inf.txt” file has only one theta value (at 0.0) saved. The **FisherInf** command allows the number of points and the symmetric minimum/maximum values of those points to be modified. For instance, if information values were desired for 81 points with theta values ranging from -4.0 to 4.0, the command to obtain this output, in conjunction with **SaveInf = Yes**;, would be **FisherInf = 81, 4.0**;; with the total number of points listed first and the maximum theta value to be used listed after comma. Using the **PriorInf** command, the user controls if

flexMIRT<sup>®</sup> includes the contribution from the prior in the test information output - the default setting of **Yes** includes the prior contribution.

Note that for multidimensional models, flexMIRT<sup>®</sup> will try to print, by default, the trace of the Fisher information matrix calculated at every grid point as defined by the direct product of the points specified in the **FisherInf** command. For high dimensional models, this is not advisable. If the **logDetInf = Yes;** command is used, the information will be taken as the log determinant of the Fisher information matrix, rather than the trace.

The **Quadrature** command, which can modify the number of quadrature points used in the E step of the EM algorithm, is specified in a similar fashion as the **FisherInf** command. The default quadrature rule has 49 rectangular points over -6 to +6.

The next several statements listed within the technical specifications deal with convergence criteria values and maximum number of iterations. **MaxE** determines the maximum allowed number of E-steps in the EM algorithm, **MaxM** the number of allowable iterations in each of the iterative M-steps. **Etol** and **Mtol** set the convergence criteria for the E- and M- steps, respectively. **SEmtol** sets the convergence criterion for the Supplemented EM algorithm used in calculating standard errors, and finally **SStol** determines the criterion for declaring two summed scores as equivalent (useful for scoring with weights).

The **Processors** statement lets the user determine how many processors (threads) an analysis can utilize. When **NewThreadModel = Yes;** a new multi-core processing model is employed. The new thread model is an updated implementation of multi-core processing that may be more efficient for very high-dimensional models with a large number of quadrature points to evaluate. The level of granularity is smaller than the classical threading model implemented in flexMIRT; a classical threading model trades more memory for speed, and this new model does not do that nearly as much, so the parallel speed up may not be as significant as the classical approach for low dimensional analyses with large N. User are encouraged to experiment with the threading options. When the **NewThreadModel** is active, **Parallelization granularity: Fine (may be more optimal for high dimensionality)** will be printed in output with other control values. **Coarse** will be printed when the original threading model is used.

Another subset of commands available in the <Options> section deals with the calculation and reporting of various GOF and local dependence (LD) indices.

**Example 7.4:** <Options> - GOF and LD Indices Commands

```
<Options >
...
GOF = Basic/Extended/Complete;
CTX2Tbl = Yes/No;
SX2Tbl = Yes/No;
MinExp = 1.0;
M2 = None/Full/Ordinal/OrdinalSW;
HabermanResTbl = Yes/No;
JSI = Yes/No;
FactorLoadings = Yes/No;
FitNullModel = Yes/No;
StartValFromNull = Yes/No;
...
```

The **GOF** statement controls the extent of GOF indices that are reported. **Basic**, **Extended**, and **Complete** are the three possible levels of GOF reporting. **Basic**, the default value, prints the  $-2 \times \log$  likelihood, the AIC/BIC values, and when appropriate, the likelihood ratio ( $G^2$ ) and Pearson  $X^2$  full-information fit statistics. The **Extended** option results in all the indices included in the **Basic** reporting, as well as the marginal fit  $X^2$  for each item and standardized LD  $X^2$  for each item pair (e.g., Chen & Thissen, 1997). The last keyword, **Complete**, prints all indices included in the **Extended** reporting and also includes, per group, the item-fit index  $S - X^2$  (Orlando & Thissen, 2000). Additionally, under **GOF = Complete;** mode, an additional statistic  $S - D^2$  is printed below the SSC to EAP conversion table. This statistic is distributed approximately as a central  $X^2$  variable, with degrees of freedom equal to [(number of summed scores - 1) - 2], and provides a test of latent variable distribution assumption.

In both the **Extended** and **Complete** GOF modes, the standardized LD  $X^2$  statistics for each item pair may be suppressed by setting the **CTX2Tbl** command to **No**. As noted in the discussion of the LD statistic values of Chen and Thissen (1997) when applied to ordinal data (i.e., Example 2.5), phi correlations are examined to determine if difference between the observed

and model implied item-correlation is positive or negative. When polytomous data are involved, there may be times that the contingency table of two items needs to be collapsed from the edges to the middle so that the cells are not too sparse. Even if collapsing occurs for the Chen-Thissen LD calculations, the phi correlations (determining the "p" or "n" after each LD value), will be based on the original (non-collapsed) tables.

The **SX2Tb1** command is invoked when the user wishes to be able to examine the detailed item fit tables used in the calculation of the  $S - X^2$  statistics. As described in Orlando and Thissen (2000), when expected frequencies for responses become small, the accuracy of the  $X^2$  approximation deteriorates. flexMIRT<sup>®</sup> employs an algorithm to evaluate the expected probabilities prior to final calculations and collapse the table when needed. The **MinExp** command allows the user to set the value at which expected counts are considered too small, (the default value for **MinExp** is set at 1.0) and when expected counts lower than the value set by **MinExp** are encountered, the summed score groups are collapsed towards the center.

The **M2** statement requests that the limited-information fit statistics  $M_2$ , first introduced by Maydeu-Olivares and Joe (2005) (**Full**) and expanded in Maydeu-Olivares, Cai, and Hernandez (2011) (**OrdinalSW**) and Cai and Hansen (2012) (**Ordinal**) be printed. The **None** option suppresses  $M_2$ . It should be noted that, regardless of **M2** request, the  $M_2$  values will not be printed unless **GOF** is set to something other than **Basic**.

The command **HabermanResTb1** is invoked when standardized residuals for each response pattern, sometimes referred to as Haberman Residuals (see Haberman, 1979), are desired. In addition to the standardized residuals, the observed and expected frequencies, as well as the associated EAPs and SD for each response pattern are also reported in the requested table.

There is an additional LD index that flexMIRT<sup>®</sup> can compute with the command **JSI**. The statement **JSI = Yes**; requests that the Jackknife Slope Index (JSI; Edwards & Cai, 2011), a recently introduced LD detection technique, be calculated. Based on the observation that locally dependent items often exhibit inflated slopes, the JSI procedure finds, for each item pair, the change in the slope parameter of item  $j$  when item  $k$  is removed from the scale. Specifically, a single JSI values is obtained by

$$JSI_{j(k)} = \frac{a_j - a_{j(k)}}{se(a_{j(k)})}, \quad (7.1)$$

where  $a$  is the IRT slope estimate,  $j$  indexes the item impacted and  $k$  indexes

the removed item and  $se(a_{j(k)})$  is the SE of the item-removed slope parameter. The resulting  $n$  item by  $n$  item matrix, with empty diagonals, is inspected by the user and item pairs with JSI values substantially larger than the other values should be noted as possibly exhibiting LD.

The **FactorLoadings** statement may be invoked when, in addition to the IRT parameters, normal metric factor loadings should be printed as part of the general output file. The factor loading values are converted from the IRT parameters, as described in Wirth and Edwards (2007), among others.

The final two statements of this group are tied to the Null (independence) model, needed for incremental fit indices. The **FitNullModel** command, when invoked with the keyword **Yes**, tells flexMIRT<sup>®</sup> to fit the Null model and print the resulting basic GOF measures in the general output file. When the fitting of the Null model is requested, it becomes possible for the Tucker-Lewis Index to be calculated. Additionally, when the zero-factor Null model is requested, the resulting parameter values from fitting this model may be used as starting values for the IRT model of interest by supplying the command, **StartValFromNull = Yes;**. Obviously, it is necessary that the Null model is requested, via the **FitNullModel** command, for the **StartValFromNull** command to be functional.

### Example 7.5: <Options> - DIF Analysis Commands

```
<Options >
...
DIF = None/TestAll/TestCandidates;
DIFcontrasts = (...);
OrthDIFcontrasts = Yes/No;
...
```

Within the <Options> section, there are three commands related to the implementation of DIF analyses. The first command is a request for a DIF analysis, **DIF**. By default, no DIF analysis is conducted, but this may be modified by the user to request either a comprehensive DIF sweep (keyword is **TestAll**), testing all items, or a focused DIF analysis which looks for DIF only in the candidate items supplied by the user (keyword is **TestCandidates**).

If the **TestCandidates** option is selected, the **DIFitems** command, found in the <Groups> section is required. When a DIF analysis is requested by invoking one of the **DIF** options, **DIFcontrasts** becomes a required command. This command is used to supply flexMIRT® with the DIF contrast matrix, which is used in constructing the contrasts among groups. The total number of contrasts is one fewer than the total number of groups being analyzed. For a 2 group DIF analyses, that means there is one allowable contrast, which will typically be of the form **DIFcontrasts** = (1 -1);. For analyses with more than 2 groups, any ANOVA contrasts can be specified. The Helmert contrast (which is orthogonal) is a reasonable choice, (e.g., for 3-groups:

```
DIFcontrasts = (
2.0 -1.0 -1.0,
0.0 1.0 -1.0);
```

When group size is highly unbalanced, **OrthDIFcontrasts** offers additional re-weighting of the contrast coefficients such that they become orthogonal with respect to the group sizes (Langer, 2008). This option is turned off by default.

Finally, there are several commands that are specific to or used primarily by the MH-RM algorithm.

**Example 7.6:** <Options> - MH-RM Specific Options

```
<Options >
...
Algorithm = BAEM/MHRM;
Imputations = 1;
Thinning = 10;
Burnin = 10;
Stage1 = 200;
Stage2 = 100;
Stage3 = 2000;
Stage4 = 0;
InitGain = 1.0;
Alpha = 1.0;
Epsilon = 1.0;
WindowSize = 3;
ProposalStd = 1.0;
ProposalStd2 = 1.0;
MCsize = 25000;
...
```

To call the MH-RM algorithm, the combination of `Mode = Calibration;`, `Algorithm = MHRM;`, and a random number seed via `RndSeed = ;` is required. All other listed commands are optional, with the default values shown above.

**Imputations** controls the number of imputations from the MH step per RM cycle. A larger value will lead to smoother behavior from the algorithm. In most cases, a value of 1 should be sufficient for obtaining accurate point estimations, however this number may be increased for better standard errors.

Thinning refers to the MCMC practice of retaining only every  $k$ th draw from a chain. Thinning is used, in part, to reduce the possible autocorrelation that may exist between adjacent draws. The **Thinning** statement sets the interval for the MH sampler - meaning if **Thinning = 10;**, every 10th draw by the sampler will be retained.

The **Burnin** statement controls the number of draws that are discarded from the start of the MH sampler. These draws are discarded to avoid using values that were sampled before the chain had fully converged to the target

distribution. `Burnin = 10`; tells flexMIRT<sup>®</sup> to discard the first 10 values obtained by the MH sampler.

`Stage1` determines the number of Stage I (constant gain) cycles. The Stage I iterations are used to improve default or user-supplied starting values for the estimation that occurs in Stage II. `Stage2` specifies the number of Stage II (Stochastic EM, constant gain) cycles. The Stage II iterations are used to further improve starting values for the MH-RM estimation that occurs in Stage III. `Stage3` sets the maximum number of allowed MH-RM cycles to be performed.

`Stage4` determines the method by which SEs are found. If `Stage4 = 0`; then SEs will be approximated recursively (see Cai, 2010b). If a non-zero value is given then the Louis formula (Louis, 1982) is used directly. If the Louis formula is to be used, the supplied value determines the number of iterations of the SE estimation routine; this number will typically need to be large (e.g., 1000 or more).

`InitGain` is the gain constant for Stage I and Stage II. If the algorithm is initially taking steps that are too large this value may be reduced from the default to force smaller steps.

`Alpha` and `Epsilon` are both Stage III decay speed tuning constants. `Alpha` is the first tuning constant and is analogous to the `InitGain` used in Stages I and II. `Epsilon` controls the rate at which the sequence of gain constants converge to zero. Specified `Epsilon` values must be in the range (0.5, 1], with a value closer to 1 indicating a faster rate of convergence to zero. `WindowSize` allows the user to set the convergence monitor window size. Convergence of the MH-RM algorithm is monitored by calculating a window of successive differences in parameter estimates, with the iterations terminating only when all differences in the window are less than 0.0001. The default value of the window size is set at 3 to prevent premature stoppage due to random variation.

`ProposalStd` and `ProposalStd2` control the dispersion of the Metropolis proposal densities for the first and second level of the specified model, respectively. If a single level model is specified, only the `ProposalStd` command will be used. **Although default values of 1.0 have been set for both of these commands, these values need to be adjusted on a case-by-case basis.** The values used will depend on the complexity of the model, the number of items, the type of items (e.g., dichotomous, polytomous), and the model fit to the items (e.g., graded, 3PLM, Nominal). The user should choose `ProposalStd` and `ProposalStd2` values so that the long-term average of the acceptance rates (which is printed for each iteration in the progress window)

for level 1 and level 2 (if applicable) are around 0.5 for lower dimensional models ( $\leq 3$  factors) and in the 0.2 - 0.3 range for higher dimensional/more complex models. Generally speaking, increasing the `ProposalStd` value will result in lowered acceptance rates while decreasing the value will result in higher acceptance rates. Users are directed to Roberts and Rosenthal (2001) for optimal scaling, choice of dispersion constants, and long-term acceptance rates of Metropolis samplers.

`MCSIZE` is the Monte Carlo size for final log-likelihood, AIC, and BIC approximations.

---

### 7.3. The Groups Section

The `<Groups>` section includes commands for specifying group names, number of items, models to calibrate, as well as more advanced features, such as commands for setting up the structure of hierarchical or multidimensional models, as well as empirical histogram priors.

**Example 7.7:** `<Groups>` - General Data/Model Descriptors

```
<Groups >
%GroupName%
File = "*.dat";
N = ?;
Varnames = vars;
Select = vars;
Missing = -9;
Ncats(vars) = ?;
Code(vars) = (?), (?);
Key(vars) = (?);
Model(vars) = ThreePL/Graded(?)/Nominal(?)/GPC(?);
```

Prior to any other group commands, a label must be assigned to a group, and it will become a token that all subsequent command statements may refer to. Even if only one group is present it must be given a label. The name provided to the group is arbitrary and at the discretion of the user; however, spaces should not be used when naming groups and the chosen name must be enclosed by percent signs.

The `File` statement is for specifying the path and file name in which item response data is held (or to be saved for `Simulation` runs). The number of

cases in the data file is set using the **N** statement. The **File** and **N** commands are listed as required here, but there is one exception: When the engine is placed under **Scoring** mode, and scoring method **SSC** is selected, the input file is optional when only the score conversion table is made without scoring individuals.

**Varnames** assigns labels to variables by which they will be referred to later in commands and in the output. Variable names are separated by commas following the equals sign of the **Varnames** statement. When variable names end with consecutive integers (e.g., v1, v2, ..., v100) a short-cut is available that can be both time and space saving. Rather than listing all the variables, a dash between the name of the first variable and the last variable (i.e., v1-v100) will indicate to flexMIRT<sup>®</sup> that all variables between v1 and 100 are desired. The optional **Select** statement allows for a subset of the variables initially read from the data file to be submitted to IRT analysis. The default for **Select** is to use all variables defined in **Varnames**.

Missing values may be represented by a specific number in data files. The default missing value is -9, but that may be modified with the **Missing** command. However, missing values may only be numeric values between -127 and 127. Common missing value symbols, such as a dot or a blank space will not be interpreted correctly by the program.

The number of valid response categories *observed in the data file* is specified with the **NCats** statements. Within the parentheses of the **NCats** statement, the names of the variables affected by the command are listed. For example, if Item 24, 28, and 32 - 40 all had four response options, this could be entered as **NCats(v24,v28,v32-v40) = 4**; in flexMIRT<sup>®</sup> syntax.

The internal representation of item response data in flexMIRT<sup>®</sup> is zero-based (i.e., response options must start at zero and go up). That being said, the program is capable of recoding the data, when old and new values are specified with the **Code** command. If the first 10 items are read from a data file that had four response options (1, 2, 3, 4), it would be necessary to recode these to zero-based, prior to fitting the IRT model. The recode statement could be **Code(v1-v10) = (1,2,3,4),(0,1,2,3);**. The original values are listed first and the values to which they should be recoded appear within the second set of parentheses. If it is necessary to also collapse the responses, so that the original codes 1 and 2 become a single response option 0 and the original 3 and 4 become a second option 1, that **Code** command could be **Code(v1-v10) = (1,2,3,4),(0,0,1,1);**. Note that recoding does not change the values of the number of response options in the data file, as defined by the **NCats**

statements.

The **Key** statement also provides a specific form of recoding that may be useful when the observed responses from multiple choice items have been read in. Rather than writing numerous **Code** statements to convert the observed responses into correct/incorrect 0/1 data, the **Key** statement allows the user to provide a scoring key for the items and instruct flexMIRT® to score the items internally. For instance, if the correct responses to Items 1 through 4 were 2, 0, 1, and 3, respectively, the **Key** statement to score these items would be **Key(v1-v4) = (2,0,1,3);**. Multiple **Code** and **Key** statements are permitted, to easily accommodate situations in which items have variable numbers of response options or, when scoring a large number of items, multiple scoring keys would result in greater user-readability of the command file by limiting command line length. It should be noted that both the **Code** and the **Key** commands are completely internal to flexMIRT® and the original data file will not be modified. Although flexMIRT® can do some basic data management, it is not built to be a data management tool. It is generally advisable to perform all necessary data cleaning and data management in a general statistical software package such as SAS or SPSS before exporting a space-, comma-, or tab-delimited file for flexMIRT® analysis.

The **Model** command follows the same general format as **Ncats**, but instead of placing a number after the equals sign, one of the four keywords for the IRT model must be chosen. For example, to fit the graded model with four categories to the hypothetical items previously mentioned, the **Model** statement is **Model(v24,v28,v32-v40) = Graded(4);**. The question mark in parentheses following the **Graded**, **Nominal**, and **GPC** keywords in Example Box 7.7 are place-holders for the number of categories the model should accommodate. When items are of mixed formats (multiple choice, free response, Likert-type scale), multiple **NCats** and **Model** statements may be needed to correctly describe the data and the chosen IRT models.

We have covered the required commands within the **<Groups>** section as well as several optional commands that are useful for selecting or modifying data. In addition to the syntax statements previously covered, there are other additional, more specialized commands which are also available.

**Example 7.8:** <Groups> - Specialized Data/Model Descriptors

```
<Groups >
...
CaseWeight = var;
Dimensions = 1;
Primary = 0;
Between = 0;
Nlevel2 = ?;
Cluster = var;
CaseID = var;
EmpHist = Yes/No;
ItemWeight(vars) = (?);
BetaPriors(vars) = ?;
Attributes = ?;
InteractionEffects = (?,?,...);
Generate = (?,?), (?,?),...;
DM = group;
Ta(vars) = Trend/Identity/(...);
Tc(vars) = Trend/Identity/(...);
DIFitems = vars;
```

The **CaseWeight** command is needed when response pattern by frequency (aka grouped) data is entered. It provides the program with the variable name for the column that contains the number of observations (or weights) for each response pattern.

The **Dimensions** statement is used to specify the total number of dimensions in the model. The default is a single-level unidimensional model (i.e., **Dimensions = 1**);). Changing the number of dimensions is the first step in setting up multidimensional and multilevel models. The **Primary** statement is used to set the number of primary, or general, dimensions/factors that will be used in a two-tier multidimensional model. In the bifactor or standard testlet model, **Primary=1**;

Several of the syntax statements in the currently discussed box are closely associated with multilevel models. To initiate the basic structure for a 2-level model, the **Between** command is used, which specifies that number of latent variables at level-2 (aka **Between**). When multilevel data are simulated, it is necessary to tell flexMIRT<sup>®</sup> how many level-2 units should be generated. This is specified by the **Nlevel2** command. This command is only necessary when

simulating multilevel data, as the program is able to determine the number of unique level-2 units from the data file, provided that the **Cluster** command is employed

**Cluster** names the variable that contains unique identifying values for the level-2 units (e.g., schools in which students are nested). **CaseID** is used to specify the variable that provides identifiers for the level-1 units.

The **EmpHist** statement is used to enable the empirical histogram (EH) characterization of the latent variable distribution, simultaneously estimated with the item parameters. The program will calibrate item parameters against the estimated EH prior, which is based on the normalized accumulated posterior densities for all the response patterns at each of the quadrature nodes, and flexMIRT<sup>®</sup> will save the EH prior in the -prm file. Additionally, for a concurrent scoring run or any future scoring runs that use the saved parameter file containing the EH values, the program will recognize that the group contains EH weights and will use them to compute EAPs or SSCs. Empirical histograms may only be used with single-level unidimensional or bifactor/testlet models. When used with a bifactor-type model, only the general dimension will be characterized using the EH.

While category weights are typically taken to be the same that the category value (e.g., a response of 3 is assigned a weight of 3), this does not have to be the case. If alternate category weights for items are desired, the **ItemWeights** statement may be used to assign such alternative weights to items. For dichotomously scored items, a statement of **ItemWeights(v1-v3) = (0.0,0.5);** could be used to assign, for variables v1 to v3, a weight of 0 to all 0 responses and a weight of 0.5 to all 1 responses. For polytomous items, the same format is holds. Suppose variables v4 to v8 had 5 possible responses categories, weights alternate to the default values of 0,1,2,3,4 could be assigned with the command, **ItemWeights(v1-v3) = (0.0,0.1,2.0,3.0,3.1);**. Due to this statement, scores of 0 are assigned a category weight of 0, 1s a weight of 0.1, 2s a weight of 2.0, all 3s given a weight of 3.0 and responses of 4 a weight of 3.1. Multiple **ItemWeights** statements are permitted, to handle situations when the number of responses options per item is variable.

**BetaPriors** is used to assign item-specific beta distribution priors on the uniquenesses; the rationale and use of applying priors to the item-uniquenesses was discussed in Bock et al. (1988) on pp. 269-270. The Beta distribution applied in flexMIRT<sup>®</sup> has an  $\alpha$  parameter set to the user-specified value and the  $\beta$  parameter fixed at 1. For example, the statement **BetaPriors(v1-v5) = 1.6;** would apply to the uniquenesses of Items 1 through 5 a prior distribution of the form Beta(1.6, 1). Multiple **BetaPriors** statements are permitted.

The next four commands are used exclusively for the fitting of cognitive diagnostic models. **Attributes** is used to set the number of main effects present in the skill space that is being assessed. This is the keyword that triggers flexMIRT<sup>®</sup> to model discrete latent variables, rather than the default continuous latent variables.

**InteractionEffects** instructs flexMIRT<sup>®</sup> to automatically generate all possible interaction effects. For instance, with 4 attributes one may specify **InteractionEffects = (2,3,4);** and, in addition to the 4 main effects, the program will also generate the 2nd, 3rd, and 4th order interaction terms of the attributes as dimensions of the model. If only the second order interaction effects are desired, those would be created by specifying **InteractionEffects = (2);**.

Similar to the **InteractionEffects** keyword, **Generate** sets up higher order interactions effects. However, rather than creating all possible interaction effects **Generate** creates only those effects specified in the statement. For example, **Generate = (3,6,7), (4,7);** is used to generate the interaction effect of attributes 3, 6, and 7 and, separately, the interaction effect of attributes 4 and 7. There is no limit to the number of interaction effects that may be specified in a **Generate** statement and the order in which interaction effects are specified does not matter.

The **DM** command stands for “diagnostic model” and is used to set the group containing observed data - this is the group to which the higher-order DM will be applied. The **DM** command is optional; one could, in principle, fit a model without the DM (higher-order) latent variables.

The commands **Ta** and **Tc** are used in conjunction with the nominal categories model, supplying contrasts for the scoring function and intercepts, respectively. The default for both **Ta** and **Tc** is a trend contrast matrix (keyword is **Trend**), but an identity matrix (keyword is **Identity**) which allows for equality constraints, or a user-supplied matrix (denoted by (...) in the syntax box) may also be employed. Multiple **Ta** and **Tc** statements may be supplied, allowing for a large degree of flexibility in the parameterization of the nominal

model.

The final command **DIFitems** in the **<Groups>** section supplies candidate DIF items. This command is required when **DIF= TestCandidates;** has been invoked in the **<Options>** section. The names of candidate items are listed across, separated by commas, (e.g., **DIFitems = v1-v4,v8;**)

## 7.4. The Constraints Section

Several of the examples have shown that there is no required statement in the **<Constraints>** section, and that it may be left blank after the (required) section header. However as models become more complex, especially if they include multiple groups, constraints will more than likely become necessary.

Listed in the left-most column of Table 7.1 are the types of constraints available. **Value** indicates the starting or fixed value of a parameter. **Free** indicates a parameter which is freely estimated, **Equal** indicates parameters which will be constrained equal, **Fix** fixes a parameter at a specific value, **Coeff** applies a provided coefficient to a given parameter - enabling proportionality restrictions, **AddConst** applies a additive constant to specified parameters, and **Prior** provides a prior distribution for that parameter.

**Table 7.1:** Constraint Commands and Keywords

Constraint Type	Parameter	Prior Distribution
Free	Intercept(?)	Normal( $\mu, \sigma$ )
Equal	Slope(?)	logNormal( $\mu, \sigma$ )
Fix	Guessing	Beta( $\alpha - 1, \beta - 1$ )
Value	ScoringFn(?)	
Coeff	Mean(?)	
AddConst	Cov(?,?)	
Prior	MainEffect(?)	
	Interaction(?,?,...)	

Presented in the second column of Table 7.1 are the keywords for parameters that may have constraints applied to them. **Intercept** refers to the location parameter. **Slope** refers to the slope/discrimination parameter. **Guessing** is the lower asymptote parameter in the 3PL model. **ScoringFn** is the scoring function contrast parameters of the nominal model. **Mean** and **Cov** refer to the mean and covariance matrix elements of the latent variable distribution.

**MainEffect** and **Interaction** are available so users may employ syntax specifications that are in keeping with the conceptualization/terminology used in cognitive diagnostic models.

The question marks in the parentheses following the parameter keywords permits the referencing of specific parameters. For example, when there are 4 latent variables in the model, **Slope(1)** refers to the slope on the first factor, **Mean(2)** refers to the mean of the second factor, and **Cov(3,2)** refers to the covariance of factors 3 and 2. When an item has 5 categories, **Intercept(2)** in a graded model or **ScoringFn(3)** in a nominal model each refers to specific item parameters.

For the **Free**, **Equal**, and **Fix** constraints, the general format for applying restrictions to the model is the same. First, the type of constraint to be applied and the group to which it is being applied are listed, followed by a comma. The items which are affected by the constraint are then specified in parentheses, followed by another comma and then the keyword for the parameter that is affected (e.g., **Slope**, **Guessing**, etc). For a unidimensional model in **Group1**, the slope parameters for v1 through v5 can be set equal using the following syntax:

```
Equal Group1, (v1-v5), Slope;
```

If there is only one group, the group name can be omitted.

```
Equal (v1-v5), Slope;
```

The following syntax fixes the second slope of (v1,v2) to 0 (the default value unless modified by **Value** statement).

```
Fix (v1,v2), Slope(2);
```

The following syntax fixes the second factor mean of **Group2** to 0 (the default value unless modified by **Value** statement).

```
Fix Group2, Mean(2);
```

The following syntax fixes the second factor variance of **Group2** to 1 (the default value unless modified by **Value** statement).

```
Fix Group2, Cov(2,2);
```

The following syntax fixes the covariance between factors 2 and 3 in **Group2** to 0 (the default value unless modified by **Value** statement).

```
Fix Group2, Cov(3,2);
```

The following syntax fixes the first scoring function contrast for (v1) to 1 (the default value unless modified by **Value** statement).

```
Fix (v1),ScoringFn(1);
```

The following syntax fixes the second scoring function contrast for (v1) to 0 (the default value unless modified by **Value** statement).

```
Fix (v1),ScoringFn(2);
```

The following syntax frees the third slope for items v1-v5 in **Group2**.

```
Free Group2,(v1-v5),Slope(3);
```

Cross-group constraints are also allowed. The basic constraint format is still used, with the addition of the constraint in the second group following a colon. In one of our multiple group examples, it was desired that the item parameters for 12 items be the same across the 2 groups. To accomplish this, the **Equal** constraints in the excerpted code below were employed.

**Example 7.9:** Constraints Across Groups, Specifying a Prior Distribution, and Freely Estimating Theta

```
...  
<Constraints >  
Free Grade3, Mean(1);  
Free Grade3, Cov(1,1);  
Equal Grade3, (v1-v12), Guessing : Grade4, (v1-v12), Guessing;  
Equal Grade3, (v1-v12), Intercept: Grade4, (v1-v12), Intercept;  
Equal Grade3, (v1-v12), Slope : Grade4, (v1-v12), Slope;  
Prior Grade3, (v1-v12), Guessing : Beta(1.0,4.0);
```

As can be seen, for the three constraints that set the item parameters across groups, the general constraint format of *type group, (items), parameter* is maintained for both groups, with a colon between the listing for the Grade 3 and Grade 4 groups. If more than two groups are to be constrained equal, additional colons, followed by the additional group information may be added. Also of note in this syntax excerpt, by default, the latent variable mean vector and covariance matrix are initially constrained to zero and identity, respectively. Using constraints, the first two lines in the example above specify that the mean and variance for the latent trait are to be freely estimated in the Grade 3 group, relative to the fixed values of the Grade 4 group which has a mean of 0 and variance of 1.

The format of the **Value**, **Coeff**, and **AddConst** constraints is similar to the previous types of constraints but with an additional piece of information. After specifying the constraint type, group, items, and parameter, a numeric value is specified as well which is separated from the parameter keyword by a comma. For example, a coefficient command could be something like, **Coeff G, (v1-v0), Slope, 1.702**; where we are applying a coefficient, in group G, for variables v1-v10, of 1.702 to the slopes, effectively incorporating the scaling constant  $D$  into the estimation.

The **Value** statement is a more general purpose constraint command. In calibration mode, it can be used to specify the values of fixed parameters or it can be used to supply starting values. In scoring mode, instead of using a parameter file (which is more convenient), one can in principle use the **Value** statement to fix item and group parameters to the calibrated estimates. In simulation mode, the **Value** statement can be used to specify generating parameters. For a **Value** constraint on one or more parameters, if the parameters are free, the numeric values supplied become starting values. If the parameters are fixed, **Value** can be used to change the default fixing values. For example, for a unidimensional model in a single group, the following syntax results in a set of constraints that fixes the item slopes to 1.0 and then frees the estimation of the variance of the latent variable, effectively creating a Rasch-type model.

```
Fix (v1-v5), Slope;
Value (v1-v5), Slope, 1.0;
Free Cov(1,1);
```

For applying prior distributions, the format is quite similar to the other constraints, but with the addition of a distribution type (and its associated pair of parameter values) after a colon. The syntax still follows the general

constraint format of *type group, (items), parameter*, with the addition of the requested prior distribution following a colon. In the across groups constraint example previously used, we specified a beta prior for the guessing parameters. The prior chosen was a beta distribution with  $\alpha - 1 = 1.0$  and  $\beta - 1 = 4.0$ . As noted in Table 7.1, the **Normal** distribution and the **logNormal** distributions are also available when assigning priors. For the **Normal**, the prior mean and standard deviation are needed. For the **logNormal**, the prior mean and standard deviation on the logarithmic scale are needed. For the **Normal** prior, if it is used on the guessing parameter, it is in fact imposed as a logit-normal prior on the logit of the guessing probability.

---

## Models

---

In this chapter, some more technical aspects of the IRT model implemented in flexMIRT<sup>®</sup> are discussed. We begin with a generalized formulation, focusing on the linear predictor portion of the model. More specific issues are discussed next.

### 8.1. A Generalized Formulation

Consider a linear predictor for a level-1 unit  $i$  (e.g., individual) in level-2 unit  $j$  (e.g., school) in a group  $g$  (e.g., country):

$$\boldsymbol{\eta}_{ijg} = \mathbf{A}_g^B \boldsymbol{\theta}_{jg}^B + \mathbf{A}_g^W \boldsymbol{\theta}_{ijg}^W,$$

where  $\boldsymbol{\eta}_{ijg}$  is an  $n_g \times 1$  vector of linear predictors of the  $n_g$  items in this group,  $\mathbf{A}_g^B$  is an  $n_g \times p$  matrix of item slopes on the  $p$  level-2 (between) latent dimensions or latent attributes (and interaction terms), and  $\mathbf{A}_g^W$  is an  $n_g \times q$  matrix of item slopes on the  $q$  level-1 (within) latent dimensions or latent attributes (and interaction terms), with  $\boldsymbol{\theta}_{jg}^B$  and  $\boldsymbol{\theta}_{ijg}^W$  being the between and within latent dimensions or attributes (and attribute interactions). Note that both  $\mathbf{A}_g^B$  and  $\mathbf{A}_g^W$  are implicitly assumed to be functions of  $d$  unknown structural parameters in  $\boldsymbol{\zeta}$ , subject to appropriate identification conditions or user specified constraints.

At the item level, the IRT model is a conditional density  $f_{\boldsymbol{\zeta}}(y_{ijk}| \eta_{ijk}) = f_{\boldsymbol{\zeta}}(y_{ijk} | \boldsymbol{\theta}_{jg}^B, \boldsymbol{\theta}_{ijg}^W)$ , where  $k = 1, \dots, n_g$  and  $y_{ijk}$  is the item response to item  $k$  from level-1 unit  $i$  in level-2 unit  $j$  in group  $g$ . Conditionally on the latent variable levels the item responses are independent such that

$$f_{\boldsymbol{\zeta}}(\mathbf{y}_{ijg} | \boldsymbol{\theta}_{jg}^B, \boldsymbol{\theta}_{ijg}^W) = \prod_{k=1}^{n_g} f_{\boldsymbol{\zeta}}(y_{ijk} | \boldsymbol{\theta}_{jg}^B, \boldsymbol{\theta}_{ijg}^W).$$

Let the distribution of  $\boldsymbol{\theta}_{ijg}^W$  be  $f_{\boldsymbol{\zeta}}(\boldsymbol{\theta}_{ijg}^W)$ , the level-1 latent variables can be

integrated out as

$$f_{\boldsymbol{\xi}}(\mathbf{y}_{ijg}|\boldsymbol{\theta}_{jg}^B) = \int f_{\boldsymbol{\xi}}(\mathbf{y}_{ijg}|\boldsymbol{\theta}_{jg}^B, \boldsymbol{\theta}_{ijg}^W) f_{\boldsymbol{\xi}}(\boldsymbol{\theta}_{ijg}^W) d\boldsymbol{\theta}_{ijg}^W.$$

Assuming further conditional independence of level-1 units on  $\boldsymbol{\theta}_{jg}^B$ , the marginal distribution of all item responses in a level-2 unit is

$$f_{\boldsymbol{\xi}}(\mathbf{Y}_{jg}) = \int \prod_{i=1}^{N_j} f_{\boldsymbol{\xi}}(\mathbf{y}_{ijg}|\boldsymbol{\theta}_{jg}^B) f_{\boldsymbol{\xi}}(\boldsymbol{\theta}_{jg}^B) d\boldsymbol{\theta}_{jg}^B,$$

where  $N_j$  is the number of level-1 units in level-2 unit  $j$ ,  $f_{\boldsymbol{\xi}}(\boldsymbol{\theta}_{jg}^B)$  is the distribution of the level-2 latent variables, and  $\mathbf{Y}_{jg} = \{\mathbf{y}_{1jg}, \dots, \mathbf{y}_{N_j jg}\}$  is the collection of item response patterns for all  $n_g$  items from the  $N_j$  level-1 units in level-2 unit  $j$  of group  $g$ .

Once the item responses are treated as fixed upon observation, the marginal likelihood of  $\mathbf{Y}_{jg}$  is defined as

$$L(\boldsymbol{\xi}|\mathbf{Y}_{jg}) = f_{\boldsymbol{\xi}}(\mathbf{Y}_{jg}).$$

Taking logs and summing over the (assumed) independent level-2 units, as well as groups, the marginal log-likelihood for the structural parameters is

$$\log L(\boldsymbol{\xi}|\mathbf{Y}_1, \dots, \mathbf{Y}_G) = \sum_{g=1}^G \sum_{j=1}^{J_g} L(\boldsymbol{\xi}|\mathbf{Y}_{jg}),$$

where  $G$  is the number of groups,  $J_g$  is the number of level-2 units in group  $g$ , and  $\mathbf{Y}_g = \{\mathbf{Y}_{1g}, \dots, \mathbf{Y}_{J_g g}\}$  is the collection of item response patterns for all level-1 and level-2 units in group  $g$ .

---

## 8.2. Item Response Models

For the  $k$ th row in  $\boldsymbol{\eta}_{ijg}$ , the linear predictor is equal to  $\eta_{ijk} = \mathbf{a}_{kg}^B \boldsymbol{\theta}_{jg}^B + \mathbf{a}_{kg}^W \boldsymbol{\theta}_{ijg}^W$ , where  $\mathbf{a}_{kg}^B$  is the set of  $p$  slopes on the between latent variables and  $\mathbf{a}_{kg}^W$  is the set of  $q$  slopes on the within latent variables.

### 8.2.1 Model with Pseudo-Guessing Parameter

This is the multilevel and multidimensional extension of the 3PL model. Let the correct/endorse (1) response probability be

$$P_{\boldsymbol{\xi}}(y_{ijk} = 1|\eta_{ijk}) = \text{guess}_{kg} + \frac{1 - \text{guess}_{kg}}{1 + \exp[-(c_{kg} + \eta_{ijk})]},$$

where  $\text{guess}_{kg}$  is the item-specific pseudo-guessing probability and  $c_{kg}$  is the item-specific intercept. Consequently,  $P_{\boldsymbol{\xi}}(y_{ijk} = 0|\eta_{ijk}) = 1.0 - P_{\boldsymbol{\xi}}(y_{ijk} = 1|\eta_{ijk})$ .

### 8.2.2 Graded Response Model

Suppose item  $k$  has  $K$  graded categories. Let the cumulative response probabilities be

$$\begin{aligned} P_{\xi}(y_{ijk} \geq 0 | \eta_{ijk}) &= 1.0, \\ P_{\xi}(y_{ijk} \geq 1 | \eta_{ijk}) &= \frac{1}{1 + \exp[-(c_{kg,1} + \eta_{ijk})]}, \\ &\vdots \\ P_{\xi}(y_{ijk} \geq K-1 | \eta_{ijk}) &= \frac{1}{1 + \exp[-(c_{kg,K-1} + \eta_{ijk})]}, \\ P_{\xi}(y_{ijk} \geq K | \eta_{ijk}) &= 0.0, \end{aligned}$$

where the  $c$ 's are item intercepts and the boundary cases are defined for consistency. Then the category response probabilities are

$$P_{\xi}(y_{ijk} = l | \eta_{ijk}) = P_{\xi}(y_{ijk} \geq l | \eta_{ijk}) - P_{\xi}(y_{ijk} \geq l+1 | \eta_{ijk}).$$

### 8.2.3 Nominal Categories Model

The nominal model fit by flexMIRT<sup>®</sup> is the multilevel extension of the reparameterized nominal model due to Thissen et al. (2010). Suppose item  $k$  has  $K$  nominal categories. Category  $l$ 's response probability is defined as

$$P_{\xi}(y_{ijk} = l | \eta_{ijk}) = \frac{\exp(s_{kg,l}\eta_{ijk} + c_{kg,l})}{\sum_{m=0}^{K-1} \exp(s_{kg,m}\eta_{ijk} + c_{kg,m})},$$

where  $s_{kg,l}$  is the scoring function value for category  $l$  and  $c_{kg,l}$  is the intercept value for category  $l$ . Dropping the  $kg$  subscript for a minute, Thissen et al. (2010) pointed out that for identification, the following restrictions should be in place:  $s_0 = 0$ ,  $s_{K-1} = K-1$ ,  $c_0 = 0$ . This can be accomplished by reparameterization. Let

$$\mathbf{s} = \begin{pmatrix} s_0 \\ \vdots \\ s_{K-1} \end{pmatrix} = \mathbf{T}_a \begin{pmatrix} 1 \\ \boldsymbol{\alpha} \end{pmatrix}, \text{ and } \mathbf{d} = \begin{pmatrix} c_0 \\ \vdots \\ c_{K-1} \end{pmatrix} = \mathbf{T}_c \boldsymbol{\gamma}.$$

The vector  $\boldsymbol{\alpha}$  is a  $(K-2) \times 1$  vector of scoring function contrasts that defines the ordering of categories, and  $\boldsymbol{\gamma}$  is a  $(K-1) \times 1$  vector of intercept contrasts.

The matrices  $\mathbf{T}_a$  and  $\mathbf{T}_c$  are fixed  $K \times (K - 1)$  matrices of contrast coefficients. By an appropriate choice of  $\mathbf{T}$ , the identification restrictions will be automatically satisfied. Thissen et al. (2010) propose the use of the following linear-Fourier contrast matrix

$$\mathbf{T} = \begin{pmatrix} 0 & 0 & \cdots & 0 \\ 1 & t_{2,2} & \cdots & t_{2,(K-1)} \\ 2 & t_{3,2} & \cdots & t_{3,(K-1)} \\ \vdots & \vdots & & \vdots \\ K-1 & 0 & \cdots & 0 \end{pmatrix},$$

where a typical element  $t_{l,m}$  for  $l = 1, 2, \dots, K$  and  $m = 1, 2, \dots, K - 1$  takes its value from a Fourier sine-series:

$$t_{l,m} = \sin \left\{ \frac{\pi(l-1)(m-1)}{K-1} \right\}.$$

For instance, for  $K = 4$ , the contrast matrix is

$$\mathbf{F} = \begin{pmatrix} 0 & 0 & 0 \\ 1 & .866 & .866 \\ 2 & .866 & -.866 \\ 3 & 0 & 0 \end{pmatrix},$$

and for  $K = 5$ , the contrast matrix is

$$\mathbf{F} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 1 & .707 & 1 & .707 \\ 2 & 1 & 0 & -1 \\ 3 & .707 & -1 & .707 \\ 4 & 0 & 0 & 0 \end{pmatrix}$$

For the nominal model, one can verify that identification conditions are all satisfied. For the GPC model, all the  $\alpha$  contrasts are set to zero.

# Appendices

## APPENDIX A

---

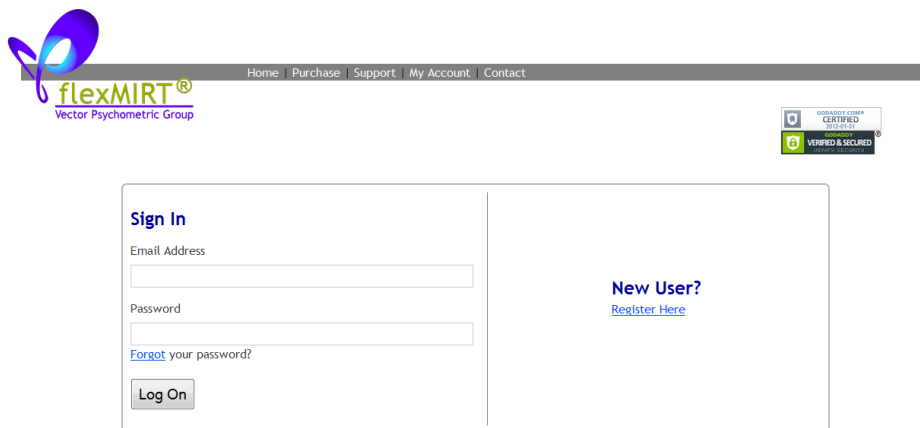
### Quick-Start Installation and User Guide

---

This guide is designed to provide an overview for installing flexMIRT<sup>®</sup> from the webpage and initializing the program for use.

#### A.1. Installing flexMIRT

To obtain the flexMIRT<sup>®</sup> installer, you must first register for an account on the flexMIRT<sup>®</sup> website. The registration page may be found by selecting the My Account link in the flexMIRT<sup>®</sup> website banner or visited directly at <https://flexmirt.vpgcentral.com/Account/LogOn>. From your account page, you will be able change your password, view your current flexMIRT<sup>®</sup> license and expiration date information, manage licenses on your various computers (with a purchased license users are allowed up to 3 separate installs of the program), as well as make payments for new or renewed licenses.



The screenshot shows the flexMIRT website interface. At the top left is the flexMIRT logo with the text "Vector Psychometric Group". To the right of the logo is a navigation bar with links: Home | Purchase | Support | My Account | Contact. Further right are two security badges: "SECURITY CERTIFIED" and "VERIFIED & SECURED". The main content area is divided into two sections. The left section is titled "Sign In" and contains fields for "Email Address" and "Password", a link for "Forgot your password?", and a "Log On" button. The right section is titled "New User?" and contains a link for "Register Here".

After selecting Register Here, you will be taken to the registration page and asked to enter your name, email address, and any affiliation information you would like to supply.

## Create a New Account

### Account Information

Name

Email address (You will use this to verify your account and login)

Institution

Department

Once you have submitted this information via the Register button, an email confirming your registration information (including the initial password for your flexMIRT® account) will be sent to the address you provided. The confirmation email will originate from [info@flexmirt.com](mailto:info@flexmirt.com) - please add this email address to your safe list and, if you have not received an email after 10 minutes, check that the email has not been redirected to your spam folder.

With the password obtained from the email, you may now log in to your flexMIRT® account. You may change your password using the Change Password link in the upper right hand corner. Before you are able to download the installer, you must accept the license agreement, which can be accessed by clicking the link in the green box.



#### Account Details

flexmirt@vpgcentral.com  
Jane Doe  
Vector Psychometric Group, LLC - Psychometrics

#### License Options

**You have a new license agreement which must be signed.**  
[Click here to Agree to the new license.](#)

#### Change license Type

Trial

You will be taken to the End User License Agreement. Once you have reviewed the agreement, there is an “I Agree” check box at the bottom of the page which you must endorse to continue. After you have accepted the license agreement, you will be able to access the flexMIRT® installer via the newly available Download Software link.



#### Account Details

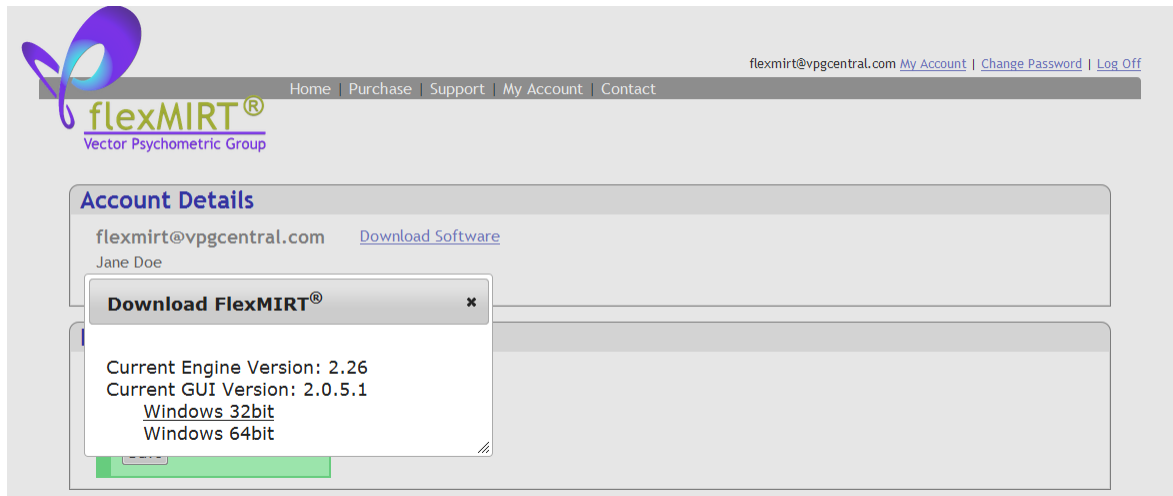
flexmirt@vpgcentral.com [Download Software](#)  
Jane Doe  
Vector Psychometric Group, LLC - Psychometrics

#### License Options

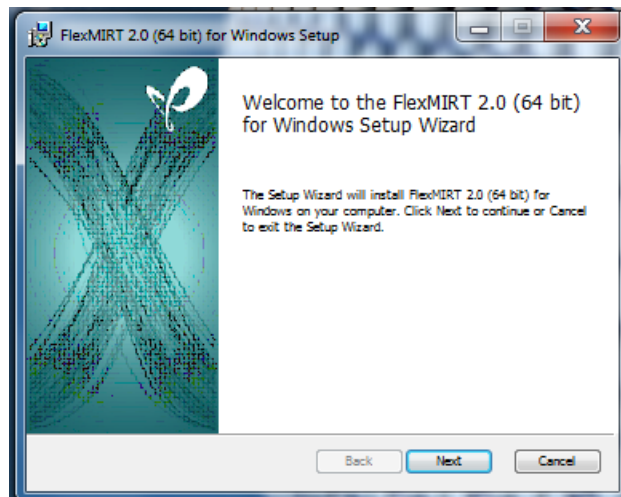
##### Select a license Type:

Trial

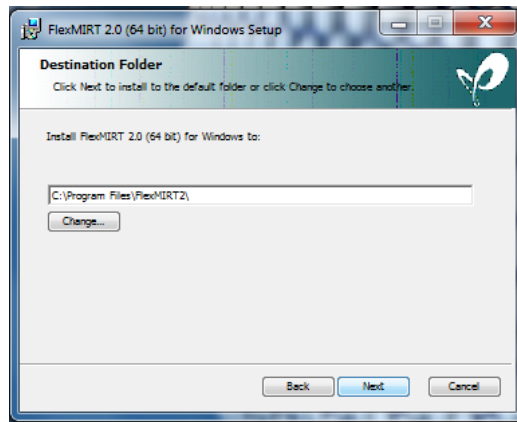
After selecting Download, a pop-up will appear where you will select the version of flexMIRT® appropriate for your computer. The flexMIRT® installer should be saved to a folder where you will be able to locate it later.



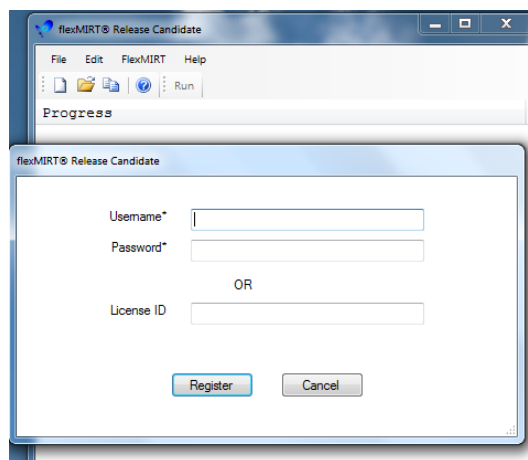
Locate the downloaded installer (flexmirtInstaller) and double-click to activate it. NOTE: IT MAY BE NECESSARY FOR SOME USERS, ESPECIALLY THOSE WITHOUT ADMINISTRATIVE PRIVILEGES, TO RIGHT-CLICK ON THE INSTALLER AND SELECT "Run as Administrator" FOR THE PROGRAM TO FUNCTION PROPERLY ONCE IT HAS BEEN INSTALLED.



The second step of the installer will ask which folder you wish to install the program into. If you would like the program installed somewhere other than the default location, this can be accomplished by selecting the “Change” button and directing the installer to the desired folder location.



Follow the remaining steps, as directed by the installer, until the program indicates that it has finished installing. Once flexMIRT<sup>®</sup> have been installed, a shortcut will appear on the desktop. Double-click this to initiate the program. On the first start-up flexMIRT<sup>®</sup> will open a pane where you must register the software; if you opt to cancel at this point you will receive a warning message and flexMIRT<sup>®</sup> will completely shut down. You may either supply the flexMIRT<sup>®</sup> account username and password you set up earlier or may copy the license code from your My Account webpage into the appropriate box.

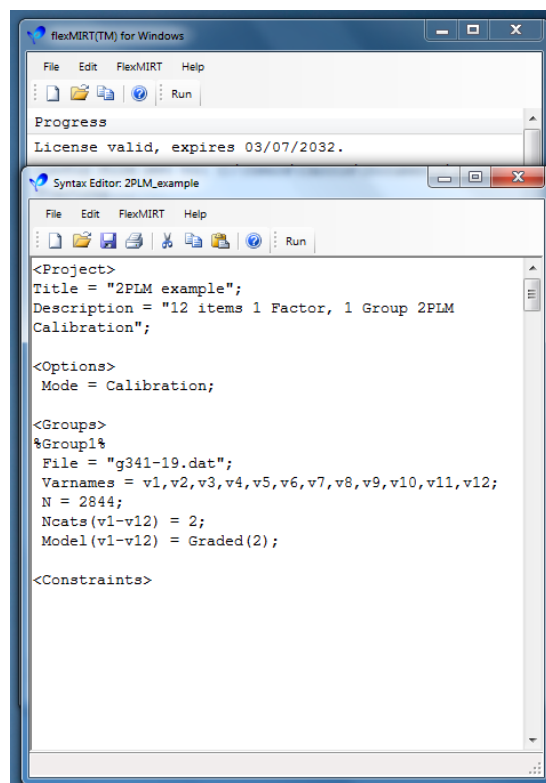


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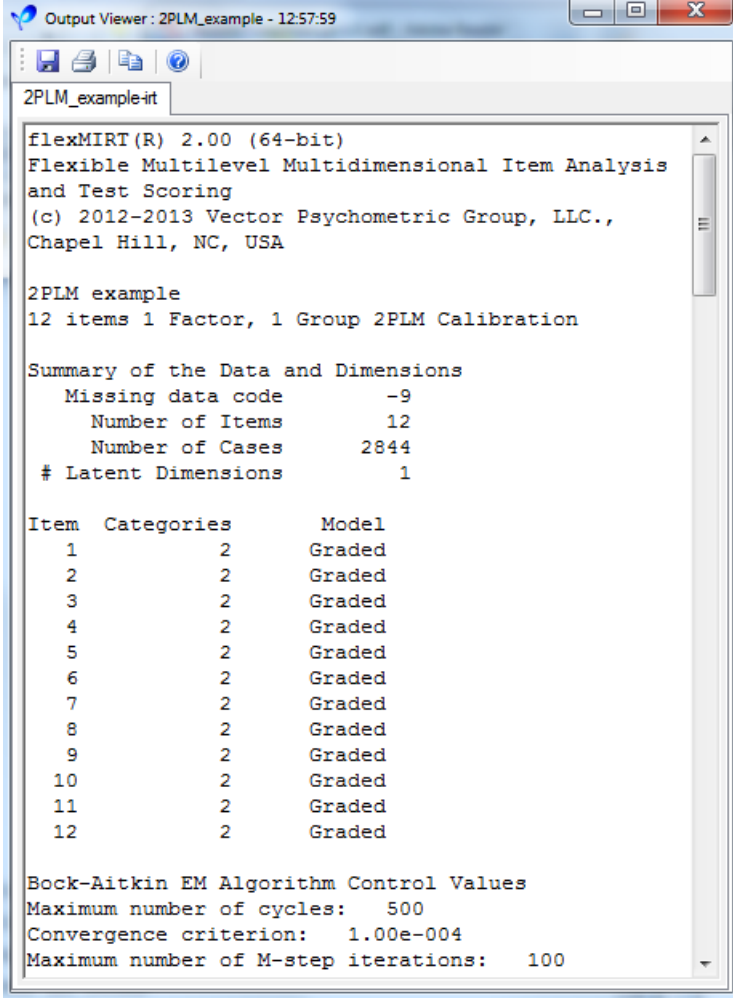
## A.2. Using flexMIRT

With the software registered, you are now able to conduct analyses. The flexMIRT<sup>®</sup> Support page (<https://flexmirt.vpgcentral.com/Support>) has the User's Manual and numerous example syntax command files with accompanying datasets, to help you become acquainted with the types of analyses that may be conducted and how command files should be structured. By selecting "New" under "File" or using the "New" icon, a command file containing necessary statements for a basic analysis opens. Once you have modified this command file, it must be saved prior to being able to submit the analysis to flexMIRT<sup>®</sup>; the "Run" button is disabled until the file has been saved to prevent the example code from being over-written.

To provide a brief tutorial, we will focus on the first example, found in the folder Example 2-1. Existing files are opened using the Open command under the File option of the engine viewing pane, which is the pane opened when the program is first initiated. Once the desired flexMIRT<sup>®</sup> command file is located and selected, it will be opened in a separate pane, labeled Syntax Editor: with the file name listed after.



The command file is submitted for analyses by clicking the “Run” button on the far right of the icon toolbar. flexMIRT® will print its progress (e.g., EM iteration number, etc.) in the engine pane and, when complete, will open the Output Viewer pane, which will contain all results. Additionally, the results are written to an external text file, which may be opened at a later time with flexMIRT® or a text editor such as Notepad.



The screenshot shows a window titled "Output Viewer : 2PLM\_example - 12:57:59". The window contains the following text:

```

flexMIRT(R) 2.00 (64-bit)
Flexible Multilevel Multidimensional Item Analysis
and Test Scoring
(c) 2012-2013 Vector Psychometric Group, LLC.,
Chapel Hill, NC, USA

2PLM example
12 items 1 Factor, 1 Group 2PLM Calibration

Summary of the Data and Dimensions
  Missing data code      -9
  Number of Items        12
  Number of Cases        2844
  # Latent Dimensions     1

Item  Categories      Model
  1         2         Graded
  2         2         Graded
  3         2         Graded
  4         2         Graded
  5         2         Graded
  6         2         Graded
  7         2         Graded
  8         2         Graded
  9         2         Graded
 10         2         Graded
 11         2         Graded
 12         2         Graded

Bock-Aitkin EM Algorithm Control Values
Maximum number of cycles:   500
Convergence criterion:    1.00e-004
Maximum number of M-step iterations:  100

```

## APPENDIX B

---

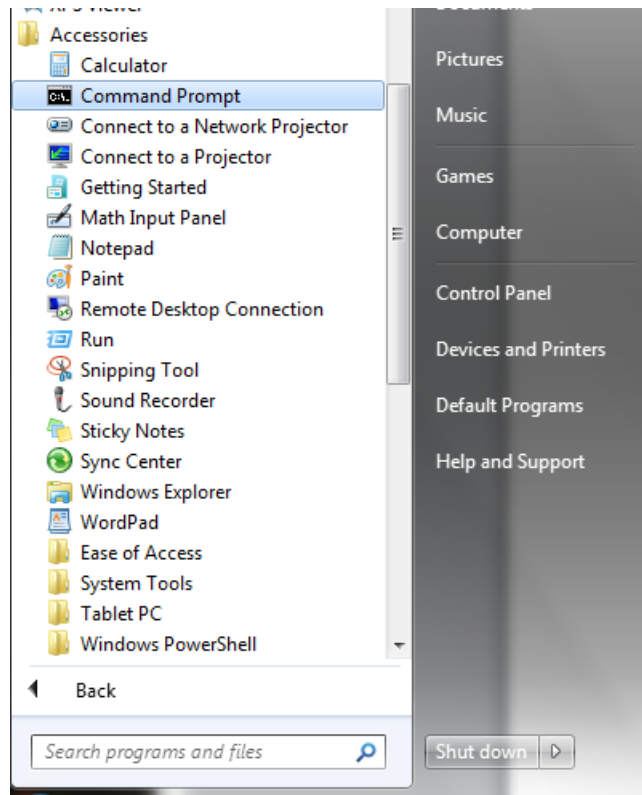
### flexMIRT via the Command Line Interface

---

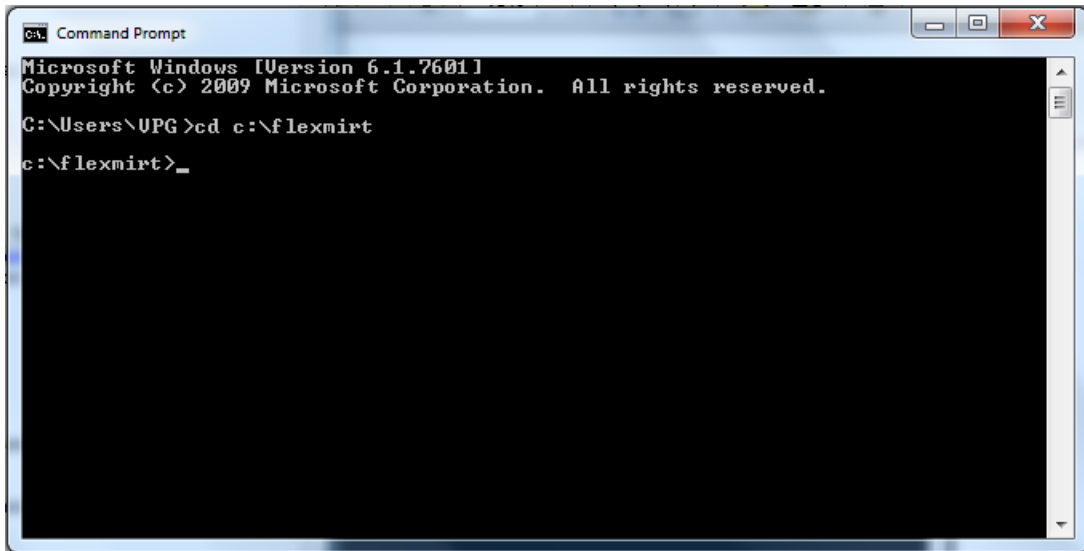
This appendix provides a brief overview of using flexMIRT<sup>®</sup> via the Command Prompt, which may be useful to those wishing to batch submit a large number of files or to control flexMIRT<sup>®</sup> via another program (e.g., R).

#### B.1. flexMIRT via the CLI - single file run

Begin by accessing the Command Prompt via the Windows Start button→ All Programs→ Accessories→ Command Prompt



You will now need to direct the CLI to the folder where the WinFlexMIRT executable is found. This is done using standard “change directory” commands. For example, if flexMIRT® was located in the folder “C:\flexmirt”, you would type “cd C:\flexmirt” where “cd” stands for “change directory” and the remaining text is the path to the folder.

A screenshot of a Windows Command Prompt window. The title bar reads "C:\ Command Prompt". The window content shows the following text: "Microsoft Windows [Version 6.1.7601] Copyright (c) 2009 Microsoft Corporation. All rights reserved. C:\Users\UPG>cd c:\flexmirt c:\flexmirt>\_". The prompt is at the end of the line "c:\flexmirt>\_".

```
C:\ Command Prompt
Microsoft Windows [Version 6.1.7601]
Copyright (c) 2009 Microsoft Corporation. All rights reserved.
C:\Users\UPG>cd c:\flexmirt
c:\flexmirt>_
```

If a correct location has been specified, the working directory will be changed, which can be seen in the above image; otherwise, an error message of “The system cannot find the path specified” will be reported.

Once the CLI has been pointed to the folder that contains the WinFlexMIRT executable, all that remains is to supply the command file name with the appropriate WinFlexMIRT call.

If the command file is found in the same folder as the executable, the call is of the form:

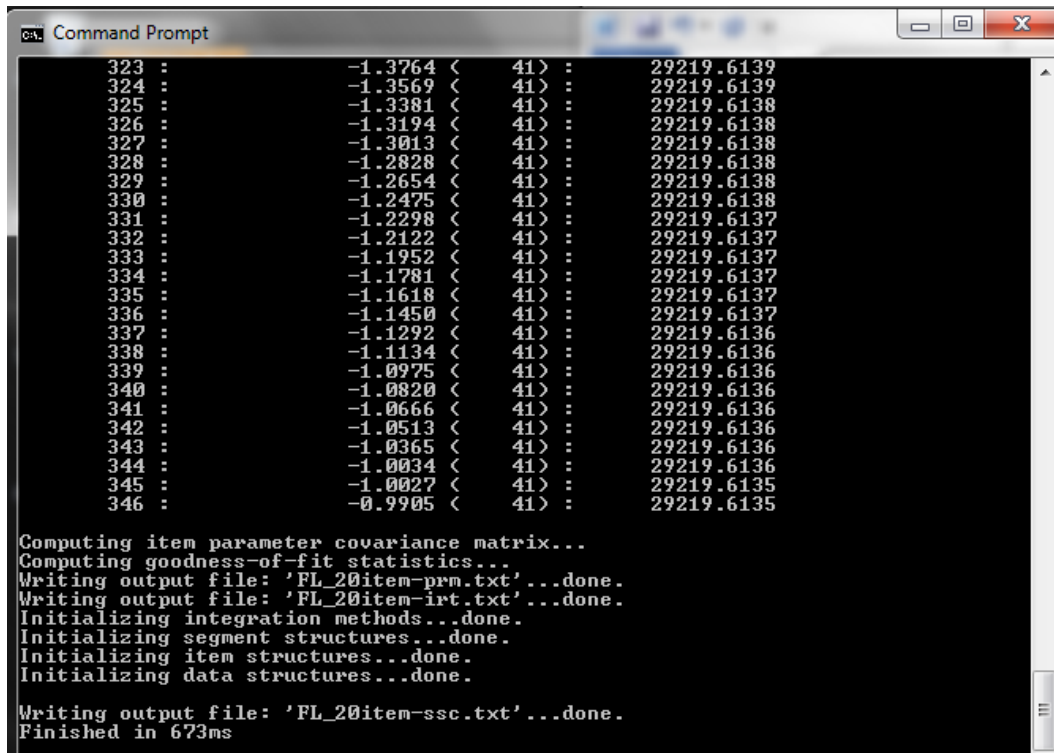
```
WinflexMIRT.exe -r mysyntax.flexmirt
```

where the “-r” tells WinFlexMIRT to run in the CLI and `mysyntax.flexmirt` is the command file you wish to run.

If the command file is found in a different folder, or has spaces in the file name, then the full path to the command file must be given in quotation marks. For example, if we wished to submit the command file “FL\_20item.flexmirt” for analysis and it is located in the folder “C:\Users\VPG\Documents\FL analyses\Final scale\”, the WinFlexMIRT call would be:

```
WinflexMIRT.exe -r "C:\Users\VPG\Documents\FL analyses\Final
scale\FL_20item.flexmirt"
```

Once a command file has been successfully submitted, the same progress information that is shown in the engine pane of the flexMIRT<sup>®</sup> graphical user interface will be printed in the CLI.



```

323 :      -1.3764 <  41> :      29219.6139
324 :      -1.3569 <  41> :      29219.6139
325 :      -1.3381 <  41> :      29219.6138
326 :      -1.3194 <  41> :      29219.6138
327 :      -1.3013 <  41> :      29219.6138
328 :      -1.2828 <  41> :      29219.6138
329 :      -1.2654 <  41> :      29219.6138
330 :      -1.2475 <  41> :      29219.6138
331 :      -1.2298 <  41> :      29219.6137
332 :      -1.2122 <  41> :      29219.6137
333 :      -1.1952 <  41> :      29219.6137
334 :      -1.1781 <  41> :      29219.6137
335 :      -1.1618 <  41> :      29219.6137
336 :      -1.1450 <  41> :      29219.6137
337 :      -1.1292 <  41> :      29219.6136
338 :      -1.1134 <  41> :      29219.6136
339 :      -1.0975 <  41> :      29219.6136
340 :      -1.0820 <  41> :      29219.6136
341 :      -1.0666 <  41> :      29219.6136
342 :      -1.0513 <  41> :      29219.6136
343 :      -1.0365 <  41> :      29219.6136
344 :      -1.0034 <  41> :      29219.6136
345 :      -1.0027 <  41> :      29219.6135
346 :      -0.9905 <  41> :      29219.6135

Computing item parameter covariance matrix...
Computing goodness-of-fit statistics...
Writing output file: 'FL_20item-prm.txt'...done.
Writing output file: 'FL_20item-irt.txt'...done.
Initializing integration methods...done.
Initializing segment structures...done.
Initializing item structures...done.
Initializing data structures...done.

Writing output file: 'FL_20item-ssc.txt'...done.
Finished in 673ms
```

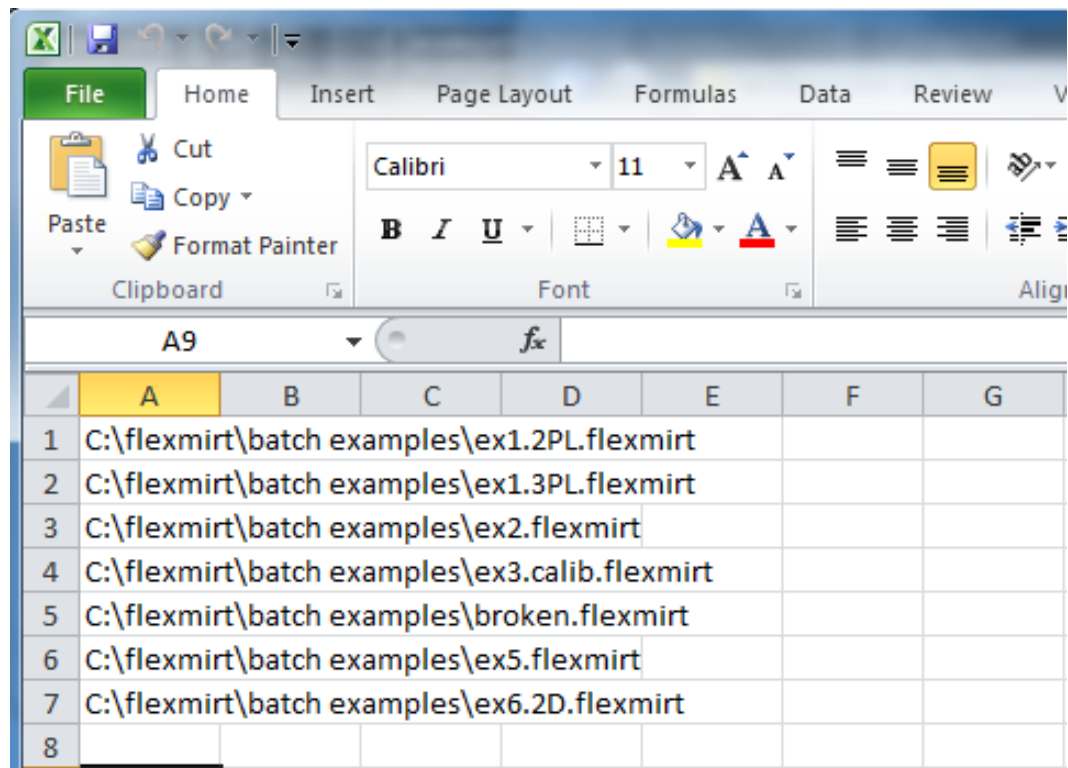
When the run has completed successfully, the output files may be found in the folder in which the command file is located.

---

## B.2. flexMIRT via the CLI - batch mode

A newer feature in flexMIRT® is the ability to submit multiple syntax files via a single batch file. The feature is only available in software releases after December 1, 2012. If you are using a version of flexMIRT® 1.0 released prior to that date, you will need to download the latest version of flexMIRT® from the website and reauthenticate your license to have access to batch mode processing.

The batch file consists of a single column of comma separate values in which the user provides the locations and file names of the command files to be run. The flexMIRT® syntax files in the batch file must ALWAYS be referenced with the full path. Below is an example batch file, saved as a CSV in Excel.



	A	B	C	D	E	F	G
1	C:\flexmirt\batch examples\ex1.2PL.flexmirt						
2	C:\flexmirt\batch examples\ex1.3PL.flexmirt						
3	C:\flexmirt\batch examples\ex2.flexmirt						
4	C:\flexmirt\batch examples\ex3.calib.flexmirt						
5	C:\flexmirt\batch examples\broken.flexmirt						
6	C:\flexmirt\batch examples\ex5.flexmirt						
7	C:\flexmirt\batch examples\ex6.2D.flexmirt						
8							

After directing the CLI to the folder that contains the WinFlexMIRT executable (see above), the batch mode is called using:

```
WinflexMIRT.exe -b batch.csv
```

where the “-b” tells WinFlexMIRT to run in the CLI batch mode and `batch.csv` is the name of the batch file. If the batch file is in a different directory than the flexMIRT® executable or contains spaces in the file name, the full path and file name should be provided and enclosed in quotes, as was shown with the single-run command line run.

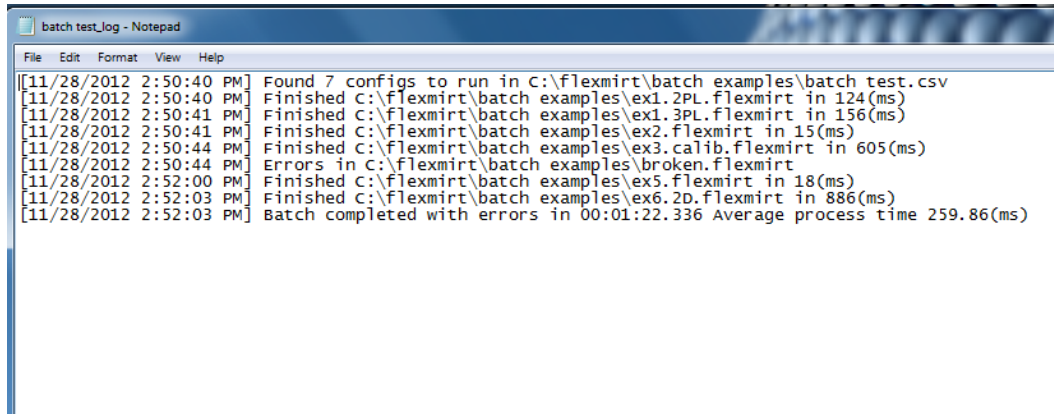
Once the batch file has been submitted, progress will be printed. The number of files found in the batch file is first reported and then the progress for individual runs starting and finishing will be given. If flexMIRT® is unable to complete a run successfully, the fact that an error was detected in the run will also be reported. As can be seen, the file “broken.flexmirt” did not complete and that fact is reported here quite plainly.

```
C:\flexmirt>winflexmirt.exe -b "C:\flexmirt\batch examples\batch test.csv"
License valid, expires 03/07/2032.
Found 7 configs to run in C:\flexmirt\batch examples\batch test.csv
Finished C:\flexmirt\batch examples\ex1.2PL.flexmirt in 124(ms)
Finished C:\flexmirt\batch examples\ex1.3PL.flexmirt in 156(ms)
Finished C:\flexmirt\batch examples\ex2.flexmirt in 15(ms)
Starting 4 of 7: C:\flexmirt\batch examples\ex3.calib.flexmirt
Starting 5 of 7: C:\flexmirt\batch examples\broken.flexmirt
Errors in C:\flexmirt\batch examples\broken.flexmirt

Starting 6 of 7: C:\flexmirt\batch examples\ex5.flexmirt
Starting 7 of 7: C:\flexmirt\batch examples\ex6.2D.flexmirt
Finished C:\flexmirt\batch examples\ex6.2D.flexmirt in 886(ms)
Batch completed with errors in 00:01:22.336 Average process time 259.86(ms)
log saved here:
C:\flexmirt\batch examples\batch test_log.txt
C:\flexmirt>_
```

Once flexMIRT® has processed all of the syntax files named in the batch file, it will report that it has finished the batch and print the total and average processing time per file. It will also give the name and location for a summary log of the batch run - the log file will be located in the same directory as the batch file and will be named with the batch file name and a “\_log.txt” suffix.

The batch log from the example batch is presented here and, as in the command prompt screen, shows that broken.flexmirt was not completed due to errors and the run time for each successfully completed syntax file and the batch as a whole.



```
batch test_log - Notepad
File Edit Format View Help
[11/28/2012 2:50:40 PM] Found 7 configs to run in C:\flexmirt\batch examples\batch test.csv
[11/28/2012 2:50:40 PM] Finished C:\flexmirt\batch examples\ex1.2PL.flexmirt in 124(ms)
[11/28/2012 2:50:41 PM] Finished C:\flexmirt\batch examples\ex1.3PL.flexmirt in 156(ms)
[11/28/2012 2:50:41 PM] Finished C:\flexmirt\batch examples\ex2.flexmirt in 15(ms)
[11/28/2012 2:50:44 PM] Finished C:\flexmirt\batch examples\ex3.calib.flexmirt in 605(ms)
[11/28/2012 2:50:44 PM] Errors in C:\flexmirt\batch examples\broken.flexmirt
[11/28/2012 2:52:00 PM] Finished C:\flexmirt\batch examples\ex5.flexmirt in 18(ms)
[11/28/2012 2:52:03 PM] Finished C:\flexmirt\batch examples\ex6.2D.flexmirt in 886(ms)
[11/28/2012 2:52:03 PM] Batch completed with errors in 00:01:22.336 Average process time 259.86(ms)
```

The typical output files for the individual runs submitted in batch mode will be in the same folder as the corresponding \*.flexmirt syntax command file, not with the batch file.

---

### Plotting

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In unidimensional IRT, graphs (e.g., tracelines and information curves) form an integral component of item analysis in practice. flexMIRT<sup>®</sup> is built to handle multilevel and multidimensional IRT models. In this context, there currently exists no universally accepted consensus with regards to graphical procedures. We do realize, however, that there is a need for publication quality graphics, and we chose to provide a set of R programs. We implemented the provided graphing functions in R because it is incredibly powerful, allows maximal customizability, and is a free program.

While flexMIRT<sup>®</sup> does not have an internal plotting function, supplemental R code to read-in flexMIRT<sup>®</sup> parameter files and plot item characteristic curves (ICCs), test characteristic curves (TCCs), item information functions (IIFs), and test information functions (TIFs) is available on the Support webpage (<http://www.flexMIRT.com/Support>) in a folder labeled “flexMIRT plotting examples”. Please note that the provided code is usable with single factor models only.

---

#### C.1. Plotting ICCs, TCCs, and Information Functions

The provided R code reads item parameters from a flexMIRT<sup>®</sup> parameter output file (with the extension \*-prm.txt). For information functions, the code reads the needed values from a flexMIRT<sup>®</sup> information output file (which has the extension \*-inf.txt). flexMIRT<sup>®</sup> *does not generate these files by default, so it is necessary to explicitly request that the parameters and information values be saved* from the calibration run using the **SavePRM = Yes**; and the **SaveInf = Yes**; commands in the **<Options>** section, respectively. Additionally, by default the Information Function output file will only contain the Information value for a single point at the theta value of 0. To change the number of points and the range over which they are spaced, one uses the **FisherInf = 21, 3.0**; command in the **<Options>** section. This specific command requests

that information values at 21 points, spaced evenly between -3.0 and 3.0, be written to the generated \*-inf.txt file; the choice of 21 and 3.0 are used as placeholders and both values can be modified by the user.

The entirety of the example plotting code (found in the file “DrawICCs\_Infs.R” in the available “flexMIRT plotting examples” folder) is presented below. Because we don’t wish to assume that users are familiar with R, we will briefly discuss the main points of the R syntax.

### Example C.1: R plotting code

```
##calls the pre-defined plotting functions - REQUIRED AS FIRST STEP
#ALTER THE PATH SO IT DIRECTS TO WHERE flexmirt.R IS SAVED ON YOUR COMPUTER
source("C:/Users/laptop/Documents/flexMIRT/flexMIRT plotting/flexmirt.R")

##For plotting ICCs and TCCs, use the function call flexmirt.icc("parameter file name")

#PROVIDE THE FULL PATH TO THE -prm FILE TO BE PLOTTED

##1PL example
flexmirt.icc("C:/Users/laptop/Documents/flexMIRT/flexMIRT plotting/SM3-prm.txt")

##2PL example
flexmirt.icc("C:/Users/laptop/Documents/flexMIRT/flexMIRT plotting/Preschool.2PL-prm.txt")
##3PL example
flexmirt.icc("C:/Users/laptop/Documents/flexMIRT/flexMIRT plotting/Preschool.3PL-prm.txt")
##Nominal Model Example
flexmirt.icc("C:/Users/laptop/Documents/flexMIRT/flexMIRT plotting/PreschoolNum_Nom-prm.txt")

##Graded Example
flexmirt.icc("C:/Users/laptop/Documents/flexMIRT/flexMIRT plotting/LiberalConservative-prm.txt")
##GPC Example
flexmirt.icc("C:/Users/laptop/Documents/flexMIRT/flexMIRT plotting/LiberalConservative-GPC-prm.txt")

##For plotting IIFs AND TIFs, use the function call flexmirt.inf("inf file name")

#NOTE: THE INF PLOTTING FUNCTION PULLS INFORMATION FROM THE *.flexmirt CALIBRATION FILE
#THAT CREATED THE *.inf FILE.
##IT IS ASSUMED THAT THE *-inf FILE and the *.flexmirt FILE ARE IN THE SAME LOCATION

##information function example

flexmirt.inf("C:/Users/laptop/Documents/flexMIRT/flexMIRT plotting/LiberalConservative-inf.txt")

##information function example - multiple grp

flexmirt.inf("C:/Users/laptop/Documents/flexMIRT/flexMIRT plotting/fit2-inf.txt")
```

As may be seen, there are brief explanations provided in the R code for those less familiar with the program and its language. Although we have provided numerous examples, these are primarily to demonstrate that the provided plotting functions work with a variety of models. There are actually only two commands that are needed to plot any given set of parameters. First, R needs to load the pre-defined plotting functions we have provided - these are found in the file “flexmirt.R” and the file is loaded into R using the “source” call seen on the first non-comment line (the first line without a # sign at the beginning).

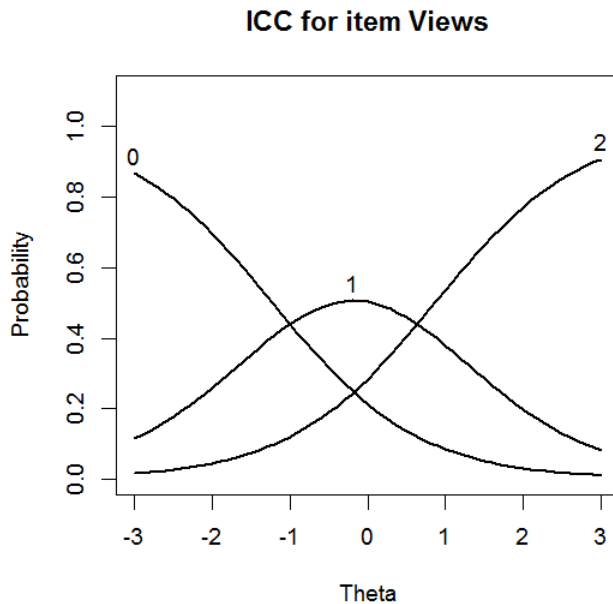
Once the functions have been loaded into R, the next step is to provide the functions with the location and file name of the values one wishes to plot. For obtaining ICCs and TCCs, the function call used is “`flexmirt.icc()`” and within the parentheses, the user supplies the \*-prm file name, with location path, that contains the parameters that will define the ICCs and TCCs. For instance, the line

```
flexmirt.icc("C:/Users/laptop/Documents/flexMIRT/flexMIRT plotting/Preschool.2PL-prm.txt")
```

calls the trace line plotting function and tells the function to create plots based on the parameter values found in the file “Preschool.2PL-prm.txt”.

The graph presented here is representative of the plot output that will be obtained from the ICC/TCC plotting function.

**Figure C.1:** GRM ICC produced by provided R code



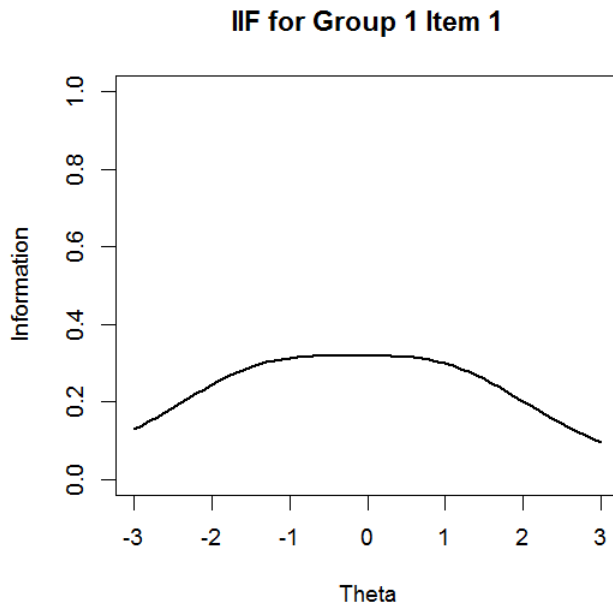
Similarly, for obtaining information function plots, the function call of “`flexmirt.inf()`” is used and the name and location of an \*-inf file is provided in the parentheses. So, the example syntax line

```
flexmirt.inf("C:/Users/laptop/Documents/flexMIRT/flexMIRT plotting/fit2-inf.txt")
```

is telling R to use the information plotting function (`flexmirt.inf`) to plot the information values found in the file “`fit2-inf.txt`.” As noted in the comments in the R code file, the information plotting function pulls information from the calibration file that created the `*-inf.txt` file. The program assumes that the `*-inf` file and the original calibration syntax file (`*.flexmirt`) will be in same folder. If, for some reason, the `*.flexmirt` file is not in the same folder as the information file, the plotting function will return an error and is unable to run.

The provided plot gives a representative of the graphics produced by the IIF/TIF plotting function.

**Figure C.2:** IIF produced by provided R code



The R programs provided are free software; you can redistribute and/or modify them under the terms of the GNU General Public License (<http://www.gnu.org/copyleft/gpl.html>) as published by the Free Software Foundation; either version 2 of the License, or (at your option) any later version. The programs are distributed in the hope that they will be useful, but WITHOUT ANY WARRANTY; without even the implied warranty of MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE. See the GNU General Public License for more details.

## APPENDIX D

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### Labeled Output Files

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Due to the lack of labels in some of the output files that flexMIRT<sup>®</sup> creates, we provide this appendix to give labeled examples of several file examples, specifically -sco and -prm files. For each example, we will provide labels at the top of each column, which would not normally appear, and a small selection of lines from the output file.

**Table D.1:** Labeled -sco file - EAP scores, 1 Factor

Grp#	flexMIRT Obs #		$\hat{\theta}$	SE( $\hat{\theta}$ )
1	1		-0.179	0.501
1	2		0.957	0.609
1	3		0.033	0.576
1	4		0.957	0.609
1	5		-0.519	0.515

**Table D.2:** Labeled -sco file - MAP scores, 1 Factor

Grp#	flexMIRT Obs #	# iterations to MAP		$\hat{\theta}$	SE( $\hat{\theta}$ )
1	1	2		-0.150	0.494
1	2	3		0.939	0.564
1	3	2		0.077	0.567
1	4	3		0.939	0.564
1	5	2		-0.489	0.518

Note: Scores from maximum likelihood (**Score** = ML;) will have a similar "# of iterations to score" column.

**Table D.3:** Labeled -sco file - EAP scores, 2 Factors

Grp#	flexMIRT Obs #	$\hat{\theta}_1$	$\hat{\theta}_2$	SE( $\hat{\theta}_1$ )	SE( $\hat{\theta}_2$ )	$\hat{\sigma}_{11}$	$\hat{\sigma}_{21}$	$\hat{\sigma}_{22}$
1	1	-0.141	-0.466	0.258	0.355	0.066705	0.001763	0.126136
1	2	0.192	1.407	0.281	0.295	0.079119	0.001470	0.087174
1	3	2.023	1.684	0.527	0.392	0.277999	0.015970	0.153786
1	4	1.311	0.840	0.334	0.290	0.111558	0.003131	0.083973
1	5	0.667	1.182	0.262	0.284	0.068758	0.001159	0.080632

**Table D.4:** Labeled -sco file - EAP scores, 2 Factors with User-supplied ID Variable

Grp#	flexMIRT Obs #	User ID	$\hat{\theta}_1$	$\hat{\theta}_2$	SE( $\hat{\theta}_1$ )	SE( $\hat{\theta}_2$ )	$\hat{\sigma}_{11}$	$\hat{\sigma}_{21}$	$\hat{\sigma}_{22}$
1	1	sub1	-0.141	-0.466	0.258	0.355	0.066705	0.001763	0.126136
1	2	sub2	0.192	1.407	0.281	0.295	0.079119	0.001470	0.087174
1	3	sub3	2.023	1.684	0.527	0.392	0.277999	0.015970	0.153786
1	4	sub4	1.311	0.840	0.334	0.290	0.111558	0.003131	0.083973
1	5	sub5	0.667	1.182	0.262	0.284	0.068758	0.001159	0.080632

**Table D.5:** Labeled -sco file - MAP scores, 2 Factors

Grp#	flexMIRT Obs #	# of Iterations to MAPs	$\hat{\theta}_1$	$\hat{\theta}_2$	SE( $\hat{\theta}_1$ )	SE( $\hat{\theta}_2$ )	$\hat{\sigma}_{11}$	$\hat{\sigma}_{21}$	$\hat{\sigma}_{22}$
1	1	3	-0.146	-0.405	0.245	0.338	0.059838	0.001456	0.114251
1	2	4	0.200	1.409	0.263	0.287	0.068942	0.001183	0.082246
1	3	5	1.838	1.643	0.494	0.368	0.244128	0.011428	0.135356
1	4	4	1.261	0.831	0.296	0.275	0.087689	0.002205	0.075827
1	5	4	0.677	1.190	0.247	0.270	0.061086	0.000914	0.072911

**Table D.6:** Labeled -prm file - GRM with 1 Factor, 1 Group

Type	Label	Grp #	# of Factors	Model	# of Cats	$c_1$	$c_2$	$c_3$	$c_4$	$a$
1	v1	1	1	1	2	5	3.24905e+000	9.45119e-001	-3.33909e+000	1.58287e+000
1	v2	1	1	1	2	5	4.35083e+000	1.79341e+000	-2.91482e+000	1.85045e+000
1	v3	1	1	1	2	5	4.04014e+000	1.60919e+000	-2.39810e+000	1.78740e+000
1	v4	1	1	1	2	5	4.11180e+000	1.64584e+000	-2.61476e+000	1.72958e+000
1	v5	1	1	1	2	5	4.34774e+000	1.71393e+000	-2.35381e+000	1.44323e+000
Type	Label	Grp#	# of Factors	Prior	$\mu_1$	$\sigma_{11}$				
0	Group1	1	1	0	0.00	1.00				

**Table D.7:** Labeled -prm file - 2PL with 6 Factors, 1 Group

Type	Label	Grp #	# of Factors	Model	# of Cats	$c$	$a_1$	$a_2$	$a_3$	$a_4$	$a_5$	$a_6$
1	v1	1	6	2	2	2.12982e+000	2.26116e+000	0.00000e+000	0.00000e+000	0.00000e+000	0.00000e+000	0.00000e+000
1	v2	1	6	2	2	3.77704e+000	2.30913e+000	2.51024e+000	0.00000e+000	0.00000e+000	0.00000e+000	0.00000e+000
1	v3	1	6	2	2	1.69155e+000	1.61782e+000	1.41918e+000	0.00000e+000	0.00000e+000	0.00000e+000	0.00000e+000
1	v4	1	6	2	2	3.76464e+000	3.86443e+000	4.89228e+000	0.00000e+000	0.00000e+000	0.00000e+000	0.00000e+000
1	v5	1	6	2	2	2.86759e+000	2.76758e+000	3.08101e+000	0.00000e+000	0.00000e+000	0.00000e+000	0.00000e+000
Type	Label	Grp#	# of Factors	Prior	$\mu_1$	$\mu_5$	$\sigma_{11}$	$\sigma_{22}$	$\sigma_{31}$	$\sigma_{32}$	$\sigma_{33}$	$\sigma_{34}$
0	Group1	1	6	0	0.00	0.00	1.00	1.00	0.00	1.00	0.00	...

**Table D.8:** Labeled -prm File - 3PL with 1 Factor, 3 groups

Type	Label	Grp #	# of Factors	Model	# of Cats	logit $g$ or $c$	$c$ or $a$	$a$
1	v1	1	1	1	2	-1.33972	-0.68782	1.301362
1	v2	1	1	1	2	-0.96814	-1.29641	1.409536
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
1	v9	1	1	1	2	-2.79306	0.730972	1.172468
1	v10	1	1	1	2	-1.65629	-0.14263	1.102673
1	v1	2	1	1	2	-1.33972	-0.68782	1.301362
1	v2	2	1	1	2	-0.96814	-1.29641	1.409536
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
1	v9	2	1	1	2	-2.79306	0.144738	1.172468
1	v10	2	1	1	2	-1.65629	-0.14263	1.102673
1	v1	3	1	2	2	-0.68782	1.301362	
1	v2	3	1	1	2	-0.96814	-1.29641	1.409536
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
1	v9	3	1	1	2	-2.79306	0.730972	1.172468
1	v10	3	1	1	2	-1.65629	-0.14263	1.102673
Type	Label	Grp#	# of Factors	Prior	$\mu_1$	$\sigma_{11}$		
0	Group1	1	1	0	0	1		
0	Group2	2	1	0	0.2	1		
0	Group3	3	1	0	-0.2	1.5		

Note that for item v1 in group 3, a different model (the 2PL, rather than the 3PL) is used. This is why the item parameter columns “logit  $g$  or  $c$ ” and “ $c$  or  $a$ ” are labeled as they are; the columns have different meanings, depending on the model fit to an item.

## APPENDIX E

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### Licensing Information

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Purchasing a license for flexMIRT<sup>®</sup> is easy. If you intend to use flexMIRT<sup>®</sup> for academic purposes, simply register at [flexmirt.vpgcentral.com/Account/LogOn](http://flexmirt.vpgcentral.com/Account/LogOn) and, once logged in, follow the instructions on-screen to download a trial version. At any point in your trial, you can log in to your account and purchase a flexMIRT<sup>®</sup> license. Your yearly subscription will begin the moment you purchase the program. Newer versions of flexMIRT<sup>®</sup> will be updated automatically on any computer with an internet connection and on which a valid copy of the program has been installed.

Each single-user license is good for three installs of flexMIRT<sup>®</sup> so you can easily work with flexMIRT<sup>®</sup> on your academic, home, and laptop computers at no additional cost. If additional installations are required, pricing information may be obtained by contacting [sales@VPGcentral.com](mailto:sales@VPGcentral.com).

Your flexMIRT<sup>®</sup> license does not automatically renew. You will be asked after your year with flexMIRT<sup>®</sup> if you would like to renew your subscription. If you desire a different subscription length, please contact [sales@VPGcentral.com](mailto:sales@VPGcentral.com) for pricing information.

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#### E.1. Commercial use of flexMIRT

This version of flexMIRT<sup>®</sup> is intended for **non-commercial use only** (as defined in the flexMIRT<sup>®</sup> End User's Licensing Agreement). If you are interested in purchasing flexMIRT<sup>®</sup> for commercial use, please contact [sales@VPGcentral.com](mailto:sales@VPGcentral.com).

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#### E.2. Classroom Discounts for flexMIRT

For educators who are interested in using flexMIRT<sup>®</sup> in their classroom, bulk educational discounts are available. Please contact [sales@VPGcentral.com](mailto:sales@VPGcentral.com) for more information.

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## Index

---

- AddConst, *see* <Constraints>, AddConst
- Algorithm, *see*
  - <Options>, Algorithm
- Alpha, *see*
  - <Options>, Alpha
- Attributes, *see*
  - <Groups>, Attributes
- BetaPriors, *see*
  - <Groups>, BetaPriors
- Between, *see* <Groups>, Between
- Between-Groups Factor, *see*
  - Multidimensional IRT,
  - Multi-level Model
- Burnin, *see* <Options>, Burnin
- Calibration
  - 1PL, 24, 25, 97, 98
    - Parameter Estimates, 26, 98
  - 2PL, 5, 6, 9, 58–60, 94, 96, 105, 112, 116, 122, 125
    - Parameter Estimates, 8, 9, 96, 107, 113
  - 3PL, 12, 40, 42, 43, 46, 48, 49
    - Parameter Estimates, 13, 14
  - Graded Response Model, 20, 49, 53, 54, 90, 94
    - Parameter Estimates, 21, 22
  - Nominal Categories, 29, 30, 32, 33, 57, 125, 156
    - Generalized Partial Credit Model, 36, 37
    - Parameter Estimates, 30, 31, 34, 59
    - Rating Scale Model, 37–39
- CaseID, *see* <Groups>, CaseID
- CaseWeight, *see*
  - <Groups>, CaseWeight
- Cluster, *see* <Groups>, Cluster
- Code, *see* <Groups>, Code
- Coeff, *see* <Constraints>, Coeff
- Cognitive Diagnostic Models, 99–103
  - C-RUM, 104, 105
  - DINA, 111, 112, 122, 134, 135
  - DINO, 115, 116
  - LCDM, 104, 111, 115, 123, 125, 132
  - Q-matrix, 103, 106, 120, 123
- Cognitive Psychometric Models, *see*
  - Cognitive Diagnostic Models
- <Constraints>, 4, 5, 12, 16, 19, 21, 24, 30, 33, 38, 39, 41, 42, 58, 157
  - AddConst, 157, 160
  - Coeff, 101, 102, 116, 117, 157, 160
  - Cov, 41, 42, 49, 50, 54, 90, 94, 158–160
  - Equal, 24, 25, 33, 38, 39, 41, 49, 94, 97, 116, 117, 121, 122, 135, 136, 157–159
  - Fix, 30, 33, 38, 39, 54, 57, 60, 90, 94, 105, 106, 112, 116, 121, 122, 125, 126, 132, 135, 157–

160  
 Free, 41, 49, 50, 54, 57, 60, 90, 94,  
 105, 106, 112, 116, 121, 122,  
 125, 126, 132, 135, 136, 157,  
 159, 160  
 Guessing, 12, 13, 25, 41, 46, 49,  
 158, 159  
 Interaction, 101, 102, 112, 116,  
 122, 125, 126, 132, 135  
 Intercept, 25, 39, 41, 49, 62, 158,  
 159  
 MainEffect, 101, 102, 105, 106,  
 112, 116, 122, 125, 126, 132,  
 135  
 Mean, 41, 42, 49, 50, 94, 158  
 Prior, 157, 161  
     Beta, 12, 41, 159, 161  
     logNormal, 12, 161  
     Normal, 12, 13, 46, 161  
 ScoringFn, 25, 30, 33, 38, 39, 57,  
 125, 126, 158, 159  
 Slope, 24, 25, 33, 38, 39, 41, 49,  
 54, 57, 60, 62, 90, 94, 97, 158,  
 159  
 Value, 61, 62, 125, 126, 157, 159,  
 160  
 Convergence Criteria, 4, 8, 30, 43, 144  
 CTX2Tbl, *see* <Options>, CTX2Tbl  
  
 Data Format, 3, 21, 153  
     Multiple Groups, 41  
     Response Pattern Data, 24, 154  
 Description, *see*  
     <Project>, Description  
 Diagnostic Classification Models, *see*  
     Cognitive Diagnostic Models  
 DIF, *see* <Options>, DIF  
  
 DIFcontrasts, *see*  
     <Options>, DIFcontrasts  
 DIFitems, *see* <Groups>, DIFitems  
 Dimensions, *see*  
     <Groups>, Dimensions  
 DM, *see* <Groups>, DM  
 DMtable, *see*  
     <Options>, DMtable  
  
 EmpHist, *see* <Groups>, EmpHist  
 Empirical Histograms, 42, 71, 72, 74,  
 155  
 Epsilon, *see*  
     <Options>, Epsilon  
 Equal, *see* <Constraints>, Equal  
 Etol, *see* <Options>, Etol  
  
 Factor Loadings, 25, 26, 55, 92  
 FactorLoadings, *see*  
     <Options>, FactorLoadings  
 File, *see* <Groups>, File  
 FisherInf, *see*  
     <Options>, FisherInf  
 FitNullModel, *see*  
     <Options>, FitNullModel  
 Fix, *see* <Constraints>, Fix  
 Fixed Effects Calibration, 97, 98  
 Fixed Theta Calibration, *see*  
     Fixed Effects Calibration  
 FixedTheta, *see*  
     <Groups>, FixedTheta  
 Free, *see* <Constraints>, Free  
  
 Generate, *see* <Groups>, Generate  
 GOF, *see* <Options>, GOF  
 Goodness of Fit (GOF), 145  
     Item-Level, 15  
     Marginal Chi-Square, 15, 22, 23

Orlando-Thissen-Bjorner  $S-X^2$ , 25–27  
 Overall, 11, 12, 35, 36, 119  
   Limited Information  $M_2$ , 33, 35, 36  
 Group Label, 4, 5  
 <Groups>, 4, 5, 12, 16, 19, 21, 24, 25, 30, 33, 37–39, 41, 43–45, 49, 54, 57, 60, 90, 94, 97  
 Attributes, 101, 105, 112, 116, 121, 122, 125, 132, 135, 156  
 BetaPriors, 41, 43, 154, 156  
 Between, 50, 51, 77, 94, 154  
 CaseID, 18, 50, 51, 154, 155  
 CaseWeight, 24, 25, 154  
 Cluster, 50, 51, 94, 154, 155  
 Code, 21, 33, 60, 151–153  
 DIFitems, 49, 148, 154, 157  
 Dimensions, 37–39, 50, 51, 54, 57, 60, 73, 77, 78, 90, 94, 105, 112, 116, 121, 122, 125, 132, 135, 154  
 DM, 101, 102, 105, 112, 116, 122, 125, 135, 156  
 EmpHist, 43, 154, 155  
 File, 5, 6, 12, 16, 19, 21, 24, 30, 33, 37–39, 41, 43, 44, 49, 50, 54, 57, 60, 62, 64, 69, 71, 73, 75, 77, 90, 94, 97, 105, 112, 116, 122, 125, 132, 135, 151  
 FixedTheta, 97  
 Generate, 101, 102, 116, 135, 136, 156  
 InteractionEffects, 101, 112, 114, 121, 122, 125, 132  
 ItemWeight, 154, 155  
 Key, 151, 153  
 Missing, 50, 151, 152  
 Model, 5, 6, 153  
   GPC(), 37  
   GPC(), 153  
   Graded(), 5, 6, 21, 24, 25, 54, 60, 62, 90, 94, 97, 105, 112, 116, 122, 125, 132, 135, 153  
   Nominal(), 30, 33, 38, 39, 57, 125, 153  
   threePL, 12, 41, 43, 49  
 N, 5, 6, 12, 16, 19, 21, 24, 30, 33, 37–39, 41, 43, 44, 49, 50, 54, 57, 60, 62, 64, 69, 71, 73, 75, 77, 78, 90, 94, 97, 105, 112, 116, 122, 125, 132, 135, 151, 152  
 Ncats, 5, 6, 12, 21, 24, 30, 33, 37–39, 41, 43, 49, 54, 57, 58, 60, 90, 94, 97, 105, 112, 116, 122, 125, 132, 135, 151, 152  
 Nlevel2, 77, 78, 154  
 Primary, 37–39, 57, 58, 60, 73, 121, 154  
 Select, 24, 25, 33, 50, 94, 97, 151, 152  
 Ta, 58, 154, 156  
   Identity, 33, 125, 132  
 Tc, 39, 58, 154, 156  
   Identity, 33  
 Varnames, 5, 6, 12, 16, 19, 21, 24, 25, 30, 33, 37–39, 41, 43–45, 49, 50, 54, 57, 60, 62, 64, 69, 71, 73, 75, 77, 90, 94, 97, 105, 112, 116, 122, 125, 132, 135, 151, 152  
 HabermanResTbl, *see*  
   <Options>, HabermanResTbl

Hierarchical Data, *see*  
     Multidimensional IRT, Multi-level Model  
 Imputations, *see*  
     <Options>, Imputations  
 Information Function, 10, 11, 42, 143, 144  
 InitGain, *see*  
     <Options>, InitGain  
 InteractionEffects, *see* <Groups>, InteractionEffects  
 ItemWeights, *see*  
     <Groups>, ItemWeights  
 JSI, *see* <Options>, JSI  
 Key, *see* <Groups>, Key  
 Local Dependence (LD), 15, 23, 28, 146  
     Jackknife Slope Index (JSI), 25, 28, 147  
     Standardized Chi-square, 15, 22, 23, 28, 145  
 logDetInf, *see*  
     <Options>, logDetInf  
 M2, *see* <Options>, M2  
 Marginal Reliability, 10, 11, 27  
 MaxE, *see* <Options>, MaxE  
 MaxM, *see* <Options>, MaxM  
 MaxMLscore, *see*  
     <Options>, MaxMLscore  
 MCsize, *see*  
     <Options>, MCsize  
 Metropolis-Hastings Robbins-Monro Algorithm, 85, 86, 92, 93  
 MinExp, *see* <Options>, MinExp  
 MinMLscore, *see*  
     <Options>, MinMLscore  
 Missing, *see* <Groups>, Missing  
 Missing Data, 3, 152  
 Mode, *see* <Options>, Mode  
 Model, *see* <Groups>, Model  
 Models, *see also* Calibration  
     3PL, 163  
     Generalized Formulation, 162  
     Graded Response Model, 164  
     Nominal Categories, 164  
 Monte Carlo Study, *see* Simulation  
 Mstarts, *see*  
     <Options>, Mstarts  
 Mtol, *see* <Options>, Mtol  
 Multidimensional IRT  
     Bifactor Model, 37–39, 56–60, 119, 122  
     Correlated Factors, 53, 54, 56, 90, 94  
     Multi-level Model, 49–52, 93–95  
     Quadrature Points, 55, 58  
 Multiple Groups  
     Calibration, 40  
     Differential Item Functioning (DIF), 45  
         Anchor Items, 49  
         Candidate DIF Items, 48  
         DIF Sweep, 45  
     Measurement Invariance, 42, 79, 160  
     Scoring, 43  
 N, *see* <Groups>, N  
 Ncats, *see* <Groups>, Ncats  
 Nested Data, *see* Multidimensional IRT, Multi-level Model

NewThreadModel, *see*  
     <Options>, NewThreadModel  
 Nlevel2, *see* <Groups>, Nlevel2  
 Non-Normal Theta Estimation, *see*  
     Empirical Histograms  
 NormalMetric3PL, *see*  
     <Options>, NormalMetric3PL  
 NumDec, *see* <Options>, NumDec  
  
 <Options>, 4, 5, 12, 16, 19, 21, 24, 30,  
     33, 41, 44, 45, 138  
     Algorithm, 87, 90, 91, 94, 97, 149  
     Alpha, 87, 88, 149, 150  
     Burnin, 87, 88, 149, 150  
     CTX2Tbl, 145  
     DIF, 148  
         TestAll, 46, 148  
         TestCandidates, 49, 148  
     DIFcontrasts, 46, 49, 148  
     DMtable, 101, 139  
     Epsilon, 87, 88, 149, 150  
     Etol, 30, 33, 37, 38, 43, 57, 58,  
         105, 112, 116, 122, 125, 132,  
         135, 136, 141, 144  
     FactorLoadings, 24, 25, 54, 55,  
         90, 92, 145, 147  
     FisherInf, 41, 42, 141, 143, 144  
     FitNullModel, 33, 35, 41, 145, 147  
     GOF, 145  
         Basic, 11, 145  
         Complete, 24–26, 145  
         Extended, 12, 15, 21, 23, 33, 41,  
             125, 132, 145  
     HabermanResTbl, 24, 25, 28, 29,  
         33, 145, 146  
     Imputations, 87, 149  
     InitGain, 87, 88, 149, 150  
     JSI, 25, 28, 146  
     JSI, 145  
     LDX2, 24  
     logDetInf, 141, 144  
     M2, 145, 146  
         Full, 33, 35, 41  
     MaxE, 105, 112, 116, 122, 125, 132,  
         135, 136, 141, 144  
     MaxM, 105, 112, 116, 122, 125, 132,  
         135, 136, 141, 144  
     MaxMLscore, 142  
     MCsize, 87, 89, 94, 149, 151  
     MinExp, 145, 146  
     MinMLscore, 142  
     Mode, 4, 5, 138, 139, 141  
         Calibration, 4, 5, 12, 21, 24,  
             30, 33, 37–39, 41, 54, 57, 60,  
             87, 90, 94, 97, 105, 112, 116,  
             122, 125, 132, 135, 149  
         Scoring, 4, 16, 19, 43–45, 138  
         Simulation, 4, 62, 64, 69, 71,  
             73, 75, 77, 138, 142  
     Mstarts, 142  
     Mtol, 30, 33, 37, 38, 57, 58, 105,  
         112, 116, 122, 125, 132, 135,  
         136, 141, 144  
     NewThreadModel, 135, 136, 141, 144  
     NormalMetric3PL, 139, 140  
     NumDec, 139  
     OrthDIFcontrasts, 148  
     Perturbation, 143  
     PriorInf, 141, 144  
     Processors, 37, 38, 54, 57, 60, 90,  
         94, 135, 141, 144  
     Progress, 41, 42, 69, 138  
     ProposalStd, 87, 89–91, 94, 149,  
         151  
     ProposalStd2, 87, 89, 94, 149, 151

Quadrature, 38, 54, 55, 57, 58, 60,  
     90, 94, 122, 141, 144  
 ReadPRMFile, 16, 19, 43–45, 64,  
     69, 71, 73, 75, 77, 141, 142  
 Rndseed, 62, 64, 69, 71, 73, 75, 77,  
     87, 90, 94, 141, 142, 149  
 SaveCov, 105, 112, 116, 122, 125,  
     132, 135, 139  
 SaveDbg, 139  
 SaveInf, 41, 42, 139, 143  
 SavePrm, 12, 33, 41, 54, 60, 90,  
     105, 112, 116, 122, 125, 132,  
     135, 139  
 SaveSco, 16, 19, 21, 24, 26, 41,  
     105, 112, 116, 122, 125, 132,  
     135, 139  
 Score, 16, 141, 142  
     EAP, 16, 41, 105, 112, 116, 122,  
         125, 132, 135, 142  
     MAP, 16, 21, 43, 44, 142  
     ML, 16, 19, 20, 142  
     SSC, 16, 24, 26, 45, 112, 114,  
         115, 142  
 SE, 141, 143  
     FDM, 143  
     Fisher, 143  
     Mstep, 143  
     REM, 105, 112, 116, 122, 125, 132,  
         135, 136, 143  
     Sandwich, 143  
     SEM, 33, 143  
     Xpd, 41, 42, 143  
 SEMtol, 141, 144  
 SlopeThreshold, 22, 139, 140  
 SmartSEM, 143  
 SStol, 141, 144  
 Stage1, 87, 88, 97, 149, 150  
 Stage2, 87, 88, 97, 149, 150  
 Stage3, 87, 88, 149, 150  
 Stage4, 87, 88, 149, 150  
 StartValFromNull, 145, 147  
 SX2Tbl, 24–26, 33, 145, 146  
 TechOut, 139  
 Thinning, 87, 149  
 WindowSize, 87, 88, 149, 150  
 OrthDIFcontrasts, *see*  
     <Options>, OrthDIFcontrasts  
 Perturbation, *see*  
     <Options>, Perturbation  
 PISA data, 49  
 Primary, *see* <Groups>, Primary  
 Prior, *see* <Constraints>, Prior  
 Prior Distribution, 5, 12, 13, 41, 42,  
     46, 160  
 PriorInf, *see*  
     <Options>, PriorInf  
 Processors, *see*  
     <Options>, Processors  
 Progress, *see* <Options>, Progress  
 <Project>, 3–5, 8, 12, 16, 19, 21, 24,  
     30, 33, 41, 44, 45, 138  
     Description, 4, 5, 8, 12, 16, 19,  
         21, 24, 30, 33, 41, 44, 45, 54,  
         60, 62, 64, 69, 71, 73, 75, 77,  
         90, 94, 97, 105, 112, 116, 122,  
         125, 132, 135, 138  
     Title, 4, 5, 8, 12, 16, 19, 21, 24,  
         30, 33, 41, 44, 45, 54, 60, 62,  
         64, 69, 71, 73, 75, 77, 90, 94,  
         97, 105, 112, 116, 122, 125,  
         132, 135, 138  
 ProposalStd, *see*  
     <Options>, ProposalStd  
 ProposalStd2, *see*  
     <Options>, ProposalStd2

Quadrature, *see*  
     <Options>, Quadrature  
 Quadrature Points, 8  
 ReadPRMFile, *see*  
     <Options>, ReadPRMFile  
 Response Pattern Standardized Residuals, 28, 29  
 Restricted Latent Class Models, *see*  
     Cognitive Diagnostic Models  
 Rndseed, *see* <Options>, Rndseed  
 SaveCov, *see* <Options>, SaveCov  
 SaveDbg, *see* <Options>, SaveDbg  
 SaveInf, *see* <Options>, SaveInf  
 SavePrm, *see* <Options>, SavePrm  
 SaveSco, *see* <Options>, SaveSco  
 Score, *see* <Options>, Score  
 Scoring, 142  
     Expected a posteriori (EAP), 16,  
         109, 129, 130, 142  
     Maximum a posteriori (MAP), 20,  
         21, 43, 44, 142  
     Maximum Likelihood (ML), 142  
     Sum Score Conversion to EAP (SSC),  
         16, 27, 44, 45, 112, 114, 115,  
         142, 152  
     Supplying Item Parameters, 16, 18,  
         20, 43, 45  
 SE, *see* <Options>, SE  
 Select, *see* <Groups>, Select  
 SEMtol, *see* <Options>, SEMtol  
 Simulation, 61  
     2PL, 72–79  
     3PL, 68–72, 74–76  
     Graded Response Model, 61–65  
     Item Parameters, 63, 64, 69, 75,  
         76, 79  
     Parameter File, 64–76  
     Value statements, 61, 62, 64  
     Multidimensional IRT  
         Bifactor Model, 72–74, 77–79  
         Multi-level Model, 77–79  
         Multiple Groups, 62, 74–79  
 SlopeThreshold, *see*  
     <Options>, SlopeThreshold  
 SmartSEM, *see* <Options>, SmartSEM  
 SStol, *see* <Options>, SStol  
 Stage1, *see* <Options>, Stage1  
 Stage2, *see* <Options>, Stage2  
 Stage3, *see* <Options>, Stage3  
 Stage4, *see* <Options>, Stage4  
 Standard Error (SE) Method  
     EM algorithm, 143  
     Empirical Cross-Product Approx-  
         imation, 8, 33, 42, 143  
     Fisher Information Matrix, 143  
     Forward Difference, 143  
     Richardson Extrapolation, 105, 143  
     Sandwich Covariance Matrix, 143  
     Supplemented EM, 33, 143  
 StartValFromNull, *see*  
     <Options>, StartValFromNull  
 SX2Tbl, *see* <Options>, SX2Tbl  
 Ta, *see* <Groups>, Ta  
 Tc, *see* <Groups>, Tc  
 TechOut, *see* <Options>, TechOut  
 Thinning, *see* <Options>, Thinning  
 Title, *see* <Project>, Title  
 Value, *see* <Constraints>, Value  
 Varnames, *see* <Groups>, Varnames  
 Weighted Data, *see* Data Format, Re-  
     sponse Pattern Data  
 WindowSize, *see*  
     <Options>, WindowSize